74. *Mordell-Weil Lattices and Galois Representation.* I

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In the subsequent notes, we announce some results on the Mordell-Weil groups of elliptic curves over a function field or, equivalently, of elliptic surfaces. The key idea is to view such a Mordell-Weil group as a *lattice* with respect to the height pairing.

First, in the part I, we formulate the basic results on the Mordell-Weil groups from this viewpoint, which leads to some new results (Theorems 1.2 and 1.4).

Then, in the part II, we apply this theory to the case of rational elliptic surfaces, and we obtain the structure theorem for the Mordell-Weil lattices of such surfaces in the most interesting case, i.e., in the case of higher rank: the Mordell-Weil lattices of rank ≥ 6 are precisely E_8, E_7^*, E_6^* or D_6^* where $E_8, E_7, \cdots$ are the root lattices and $*$ indicates the dual lattices (Theorem 2.1). As a direct consequence, we can find *very effectively* the generators of such a Mordell-Weil group (Theorem 2.2). Next we make everything more explicit in terms of the Weierstrass form (Theorem 3.2). Another key idea is the use of the specialization map to an additive fibre (Lemma 3.3). In §4, we give some examples of the elliptic surfaces of Delsarte type.

In the part III, we discuss the Galois representations arising from the Mordell-Weil lattices. We can essentially answer the problem raised by Weil and Manin ([15, p. 558], [6, Ch. 4: 23. 13]).

1. The Mordell-Weil lattices. Let k be an algebraically closed field of arbitrary characteristic. Let $K=k(C)$ be the function field of a smooth projective curve C over k. Let E be an elliptic curve defined over K, given with a K-rational point O, and let $E(K)$ denote the group of K-rational points of E, with the origin O.

We consider the associated elliptic surface $f: S \to C$ (the Kodaira-Néron model of E/K). By this we mean the following: S is a smooth projective surface defined over k and f is a morphism of S onto C such that 1) the generic fibre is E and 2) no fibres contain an exceptional curve of the first kind (i.e. a smooth rational curve with self-intersection number -1). The existence and the uniqueness, up to an isomorphism, of the Kodaira-Néron model is well-known ([4], [7], [13]).

Now the global sections of $f: S \to C$ are in a natural one-to-one correspondence with the K-rational points of E. Thus we use the same notation $E(K)$ to denote the group of sections of f. For $P \in E(K)$, (P) will denote