66. A Numerical Characterization of Ball Quotients for Normal Surfaces with Branch Loci

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(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1989)

We show that Kähler-Einstein geometry fits in with the theory of minimal models for normal surfaces with branch loci (cf. [14], [15]). One of important consequences of this is that an inequality of Miyaoka-Yau type holds for canonical normal surfaces with branch loci and with at worst log-canonical singularities and the equality characterizes ball quotients with finite volume. For details of this note, we refer to Sakai's survey article [15] on the classification of normal surfaces, Nakamura's master thesis [11] on the classification and uniformization of log-canonical surface singularities and Kobayashi's survey article [5] on uniformization of complex surfaces.

In this note, we mean by a divisor a Weil divisor, i.e., a linear combination of irreducible curves with integer coefficients. We use Mumford's intersection theory (see [10] and [15]) for $\boldsymbol{Q}$-divisors on normal surfaces. Let (V, D, p) be a pair of a germ of a normal complex surface V and a $\boldsymbol{Q}$-divisor $D=\sum_i(1-(1/b_i))D_i$ with $b_i=2, 3, \cdots, \infty$. We formally identify $\operatorname{Supp} D_i$ with a branch locus having the branch index b_i. In cace $b_i=\infty$, we consider the complement of such D_i. To understand such identification, it suffices to look at the coverings

(1) $$\boldsymbol{D} \ni z \longmapsto w=z^b \in \boldsymbol{D}$$

and

(2) $$\boldsymbol{D} \ni z \longmapsto w=\exp\left(\frac{z+1}{z-1}\right) \in \boldsymbol{D}^*,$$

where $\boldsymbol{D}$ and $\boldsymbol{D}^*$ denote the unit disk and the punctured unit disk, respectively. We say that a point p of V is a singularity of (V, D, p) if p is a singular point of V or if p is a smooth point of V and the curve $\operatorname{Supp}(D)$ has a singularity at p. Take a resolution

$$\mu: (\tilde{V}, \tilde{D}, \tilde{E}) \longrightarrow (V, D, p)$$

where $\tilde{D}$ and $\tilde{E}$ are the strict transforms of D and the exceptional set of $\mu: \tilde{V} \to V$, respectively. Let $\tilde{E}=\sum_\alpha E_\alpha$ be the decomposition of $\tilde{E}$ into irreducible components. Using Mumford's intersection theory, we define Δ by setting

(3) $$\mu^*(K_V+D)=K_{\tilde{V}}+\tilde{D}+\Delta.$$

(4) **Definition.** A singularity (V, D, p) is called *log-canonical* (resp. *log-terminal*) if there exists a good resolution μ such that $\Delta=\sum_\alpha a_\alpha E_\alpha$ with

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