64. *Topological Aspects of Conformally Flat Manifolds*

By Shigenori MATSUMOTO
Department of Mathematics, College of Science and Technology,
Nihon University

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1. Introduction. This is a research announcement concerning foundations of conformally flat manifolds. We assume throughout that M be a *closed* oriented smooth n-dimensional manifold and any map or transformation which appears in the sequal is orientation preserving.

A Riemannian manifold (M, g) is called *conformally flat* if for any $x \in M$, there exist a neighbourhood U of x and a smooth embedding $\phi: U \to S^n$ such that $\phi^* g_S = \mu \cdot g$, where g_S is the spherical metric of S^n and μ is a positive valued continuous function on U.

Recall Liouville's theorem: any (locally defined) conformal map of S^n is the restriction of a Moebius transformation, provided $n \geq 3$. Thus the above ϕ is unique up to the composition with a Moebius transformation, if $n \geq 3$. This quickly yields a system of local charts of M modelled on S^n with transition functions Moebius transformations. Further, by means of analytic continuation, we get a developing map $D: \tilde{M} \to S^n$ and a holonomy homomorphism $h: \pi_1(M) \to Mob^+(S^n)$, where $\tilde{M}$ is the universal covering of M and $Mob^+(S^n)$ is the group of all the orientation preserving Moebius transformations of S^n. They satisfy $D(\gamma x) = h(\gamma) D(x)$, where $\gamma \in \pi_1(M)$ and $x \in \tilde{M}$. The image of h is called the holonomy group and denoted by Γ.

In dimension 2, by a conformally flat structure we mean the structure given by the pair of a developing map and a holonomy homomorphism, i.e. the geometric structure known as *projective structure*.

Examples of conformally flat structures are usually constructed as follows. Let $\Gamma \subset Mob^+(S^n)$ be a discrete subgroup which acts freely and properly discontinuously on a Γ-invariant domain U of S^n. Then $M = U/\Gamma$ carries naturally a conformally flat structure. However examples are known of conformally flat manifolds whose developing maps are not covering maps. ([3], [6], [7], [9])

In § 2, given a conformally flat manifold M, we define its limit set, a subset of S^n, and give criterions for the developing map to be a covering map. In § 3, we describe conditions for M to have a finite limit set. § 4 is devoted to the study of the case where the limit set is a Cantor set.

Details including full proofs will appear elsewhere.

2. Limit set. We define the limit set of M in four different ways and show that they all coincide. Recall that Moebius transformations on S^n are extended in a canonical way to transformations on D^{n+1} and that they pre-