62. Generalized Hypergeometric Equations with Certain Finite Irreducible Monodromy Groups

By Takao SASAI
Department of Mathematics, Tokyo Metropolitan University

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1989)

In this paper we shall study the irreducibility condition for monodromy groups of generalized hypergeometric equations (say GHGE, for brevity) and determine, under a certain condition, their explicit forms when they are finite groups. Recently Beukers-Heckman [1] obtained independently the same condition ([1], Propositions 2.7 and 3.3) and determined the cases of finite monodromy groups generally by a method quite different from ours. So we shall state a remark about the latter from our standpoint.

Let us consider GHGE in the form of Okubo type (see [4]);

$$(\sharp) \qquad (tI-B)\frac{dx}{dt}=Ax,$$

where $t\in S$ (the Riemann sphere), $x={}^t(x_1, \cdots, x_n)$ is a column n-vector, I is the n by n unit matrix, B is the n by n diagonal matrix $\mathrm{diag}(0, \cdots, 0, 1)$ and A is an n by n constant matrix;

$$A=\left(\begin{array}{ccc|c} -a_1 & & 0 & 1 \\ & \ddots & & \vdots \\ 0 & & -a_{n-1} & 1 \\ \hline b_1 & \cdots & b_{n-1} & -a_n \end{array}\right)$$

with n distinct eigenvalues $-\rho_1, -\rho_2, \cdots, -\rho_n$. Moreover we assume the following;

(A) None of the quantities a_i, a_j-a_k and $\rho_l-\rho_m$ $(i, l, m=1, \cdots, n;\ j, k=1, 2, \cdots, n-1;\ j\neq k, l\neq m)$ is an integer. Moreover each ρ_j is not a positive integer.

The equation $(\sharp)$ is Fuchsian on S with three regular singular points $t=0$, 1 and ∞. From (A) there is no logarithmic solution.

Remark 1. Since $(\sharp)$ is accessory parameter free, the coefficients b_i are written in terms of a_j and ρ_k (see [4], §1). Eliminating $x_1, \cdots, x_{n-1}$ and setting $x=x_n$, we obtain

$$(b) \qquad [\delta(\delta+a_1-1)\cdots(\delta+a_{n-1}-1)-t(\delta+\rho_1)\cdots(\delta+\rho_n)]x=0,$$

where $\delta=t(d/dt)$. It is just the classical GHGE which has

$${}_nF_{n-1}\left(\begin{matrix}\rho_1, \cdots, \rho_n\,;\, t\\ a_1, \cdots, a_{n-1}\end{matrix}\right)=\sum_{k=0}^{\infty}\frac{(\rho_1)_k\cdots(\rho_n)_k}{(a_1)_k\cdots(a_{n-1})_k k!}t^k$$

as its particular solution at $t=0$, where $(\alpha)_k=\alpha(\alpha+1)\cdots(\alpha+k-1)$ (for details, see [4], §1 and §5).

We first remind Theorem 2 in [4] which was originally obtained in [3].