54. A Discrepancy Problem with Applications to Linear Recurrences. II

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This is continued from [0].

The following result gives an estimation for the discrepancy of a special s-dimensional sequence (x_n), $n=1,2,\cdots$. Let us recall the definition of the *discrepancy* $D_N(x_n)$. Generally speaking the discrepancy is a measure for the distribution behaviour of (x_n) modulo 1. More precisely put

$$A_N(x_n, I)=\text{card}\,\{n\leqq N:\{x_n\}\in I\}$$

for the number indices n such that the (componentwise) fractional part of x_n is contained in a given s-dimensional interval I. Then

$$D_N(x_n):=\sup_I\left|\frac{A_N(x_n, I)}{N}-|I|\right|,$$

where the supremum is taken over all s-dimensional subintervals I of $[0,1]^s$ with volume $|I|$. Thus, if $|I|\geqq 2D_N$, there exists an integer n with $1\leqq n\leqq N$, such that $\{x_n\}\in I$. If $D_N(x_n)$ tends to zero (for $N\to\infty$) then (x_n) is called *uniformly distributed* modulo 1 (cf. [6]).

Theorem 1. *Let $y_1,\cdots,y_s$ be a multiplicatively independent system of unimodular complex algebraic numbers and let θ_k be real numbers defined by*

$$y_k=e^{2\pi i\theta_k}\ (k=1,\cdots,s).$$

Set $\theta=(\theta_1,\cdots,\theta_s)$ and let $\omega=(\omega_1,\cdots,\omega_s)$ be an arbitrary s-tuple of real numbers. Then the discrepancy of the s-dimensional sequence $(x_n)=(n\theta+\omega)$ satisfies the estimate

$$D_N(x_n)\leqq N^{-\delta}$$

for any sufficiently large N, where $\delta(>0)$ depends only on the system $y_1,\cdots,y_s$.

Proof. Let m be an arbitrary positive integer. Then by the inequality of Erdös-Turán-Koksma (cf. [6], p. 116) we have

$$D_N(x_n)\leqq c_s\left(\frac{1}{m}+\sum_{0<\|h\|\leqq m}\frac{1}{r(h)}\left|\frac{1}{N}\sum_{n=1}^{N}e^{2\pi i\langle h,x_n\rangle}\right|\right), \tag{9}$$

where c_s is a constant depending only on the dimension s, the first sum runs through all integral lattice points $h=(h_i,\cdots,h_s)\neq(0,\cdots,0)$ with

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