50. *Regular Near-rings without Non-zero Nilpotent Elements*

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(Communicated by Shokichi IYANAGA, M. J. A., June 13, 1989)

1. Introduction. In ring theory, it is well known that regular rings without non-zero nilpotent elements are characterized in terms of quasi-ideals, that is, the following conditions on a ring R are equivalent:

(a) R is regular and has no non-zero nilpotent elements.

(b) Every quasi-ideal of R is idempotent.

(c) For any two quasi-ideals Q_1, Q_2 of R, $Q_1 \cap Q_2 = Q_1 Q_2$.

See [4, Theorem 11.5].

The purpose of this note is to characterize regular zero-symmetric near-rings without non-zero nilpotent elements, in terms of quasi-ideals. The analogy between the regular rings and near-rings without non-zero nilpotent elements is not complete.

For the basic terminology and notation we refer to [3].

2. Preliminaries. Let N be a near-ring, which always means right zero-symmetric one throughout this note.

If A, B and C are three non-empty subsets of N, then AB (ABC) denotes the set of all finite sums of the form $\sum a_k b_k$ with $a_k \in A$, $b_k \in B$ ($\sum a_k b_k c_k$ with $a_k \in A$, $b_k \in B$, $c_k \in C$). Note that $ABC = (AB)C \subseteq A(BC)$ in general.

A right N-subgroup (left N-subgroup) of N is a subgroup H of $(N, +)$ such that $HN \subseteq H$ ($NH \subseteq H$). For every subgroup H of $(N, +)$, HN is a right N-subgroup of N.

A quasi-ideal of N is a subgroup Q of $(N, +)$ such that $QN \cap QN \subseteq Q$ (see [5, Proposition 3]). Right N-subgroups and left N-subgroups are quasi-ideals. The intersection of a family of quasi-ideals is again a quasi-ideal.

A near-ring N is called regular, if for every element n of N there exists an element x in N such that $nxn = n$.

Lemma 1. *If a near-ring N is regular, then every quasi-ideal Q of N has the form $QNQ = Q$.*

Proof. Let Q be a quasi-ideal of N, that is, $QN \cap NQ \subseteq Q$. By the regularity of N, $Q \subseteq QNQ$. Moreover we have $QNQ \subseteq QN$ and $QNQ \subseteq NQ$. Hence it follows that

$$Q \subseteq QNQ \subseteq QN \cap NQ \subseteq Q.$$

Thus $Q = QNQ$.

Now we state here some known results which will be used later.