48. Zonal Spherical Functions on the Quantum Homogeneous Space $SU_q(n+1)/SU_q(n)$

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In this note, we give an explicit expression to the zonal spherical functions on the quantum homogeneous space $SU_q(n+1)/SU_q(n)$. Details of the following arguments as well as the representation theory of the quantum group $SU_q(n+1)$ will be presented in our forthcoming paper [3]. Throughout this note, we fix a non-zero real number q.

1. Following [4], we first make a brief review on the definition of the quantum groups $SL_q(n+1;C)$ and its real form $SU_q(n+1)$.

The coordinate ring $A(SL_q(n+1;C))$ of $SL_q(n+1;C)$ is the C-algebra $A=C[x_{ij};\ 0\le i,\ j\le n]$ defined by the "canonical generators" x_{ij} $(0\le i,\ j\le n)$ and the following fundamental relations:

(1.1) $$x_{ik}x_{jk}=qx_{jk}x_{ik},\qquad x_{ki}x_{kj}=qx_{kj}x_{ki}$$

for $0\le i<j\le n$, $0\le k\le n$,

(1.2) $$x_{il}x_{jk}=x_{jk}x_{il},\qquad x_{ik}x_{jl}-qx_{il}x_{jk}=x_{jl}x_{ik}-q^{-1}x_{jk}x_{il}$$

for $0\le i<j\le n$, $0\le k<l\le n$ and

(1.3) $$\det_q=1.$$

The symbol $\det_q$ stands for the *quantum determinant*

(1.4) $$\det_q=\sum_{\sigma\in S_{n+1}}(-q)^{l(\sigma)}x_{0\sigma(0)}x_{1\sigma(1)}\cdots x_{n\sigma(n)},$$

where S_{n+1} is the permutation group of the set $\{0,1,\cdots,n\}$ and, for each $\sigma\in S_{n+1}$, $l(\sigma)$ denotes the number of pairs (i,j) with $0\le i<j\le n$ and $\sigma(i)>\sigma(j)$. This algebra A has the structure of a Hopf algebra, endowed with the *coproduct* $\Delta: A\to A\otimes A$ and the *counit* $\varepsilon: A\to C$ satisfying

(1.5) $$\Delta(x_{ij})=\sum_{k=0}^{n}x_{ik}\otimes x_{kj}\quad\text{and}\quad\varepsilon(x_{ij})=\delta_{ij}\qquad\text{for } 0\le i,\ j\le n.$$

Moreover, there exists a unique conjugate linear anti-homomorphism $a\mapsto a^*: A\to A$ such that

(1.6) $$x_{ji}^*=S(x_{ij})\qquad\text{for } 0\le i,\ j\le n$$

with respect to the *antipode* $S: A\to A$ of A. Together with this **-operation*, the Hopf algebra $A=A(SL_q(n+1;C))$ defines the *-Hopf algebra $A(SU_q(n+1))$.

In what follows, we denote by G the quantum group $SU_q(n+1)$ and by K the quantum subgroup $SU_q(n)$ of $G=SU_q(n+1)$. Denote by y_{ij} $(0\le i,$

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