47. *A Note on Irreducible Representations of Profinite Nilpotent Groups*

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1. The purpose of this work is to parametrize the set of isomorphism classes of complex continuous finite dimensional irreducible representations of a profinite nilpotent group G by certain characters of the Lie ring $L(G)$ of G which is formed from the lower central series of G. Since every component $L_i(G)$ of $L(G)$ is a certain quotient of $T_i(G^{ab})$, the i-fold tensor product of $G^{ab}=G/[G,G]$, this implies that the irreducible representations of G are determined by certain characters of G^{ab}.

2. Let G be a profinite nilpotent group, and for every integer $c\geq 1$, denote by $I^c(G)$ the set of isomorphism classes of (complex continuous finite dimensional) irreducible representations of G such that their finite images are nilpotent of class c. Put

$$I(G):=\bigcup_{c\geq 1} I^c(G).$$

Denote the closed commutator subgroup of G by $[G,G]$ and put

$$G^{ab}=G/[G,G],$$

$$T_i(G^{ab})=i\text{-fold tensor product of } G^{ab},$$

$$T^c(G)=\prod_{i=1}^{c} T_i(G^{ab}),\qquad T(G)=\prod_{i\geq 1} T_i(G^{ab}).$$

For a locally compact abelian group A denote its Pontrjagin dual by $A^{\wedge}$. We shall show the substantial contents of the following statement in the sequel of the proof:

Theorem 1. *There are quotients $\bar{T}^c(G)$ and $\bar{T}(G)$ of $T^c(G)$ and $T(G)$, respectively, which are determined by certain relations between commutators of G, and surjective maps*

$$\bar{T}^c(G)^{\wedge}\longrightarrow \bigcup_{i=1}^{c} I^c(G),\qquad \bar{T}(G)^{\wedge}\longrightarrow I(G).$$

Remark. A preliminary version of this result is contained in [3], § 9, and showed on the basis of Clifford's theory (e.g. [1]-V, or [3], § 5, for the profinite case) and the results of Yamazaki [4] on projective representations of finite groups. However, we give here a different proof based on the results of Iwahori and Matsumoto [2] which shows that the maps may be considered canonically.

3. In the proof of the theorem we use the following notation. Let

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