45. On Convolution Theorems

By Shigeyoshi OWA
Department of Mathematics, Kinki University

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The object of the present paper is to prove convolution theorems for close-to-convex functions of order α and type β and convex functions of order γ, and for functions satisfying $\mathrm{Re}\{f'(z)\}>\alpha$ and convex functions.

1. Introduction. Let $\mathcal{A}$ be the class of functions of the form

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disk $\mathcal{U}=\{z: |z|<1\}$. We denote by $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ the subclasses of $\mathcal{A}$ consisting of functions which are, respectively, starlike of order α $(0\leqq\alpha<1)$ in $\mathcal{U}$ and convex of order α $(0\leqq\alpha<1)$ in $\mathcal{U}$. In particular, we write $\mathcal{S}^*(0)\equiv\mathcal{S}^*$ and $\mathcal{K}(0)\equiv\mathcal{K}$.

A function $f(z)$ belonging to the class $\mathcal{A}$ is said to be close-to-convex of order α and type β if there exists a function $g(z)$ in the class $\mathcal{K}(\beta)$ such that

$$\mathrm{Re}\left\{\frac{f'(z)}{g'(z)}\right\}>\alpha \tag{1.2}$$

for some α $(0\leqq\alpha<1)$ and for all $z\in\mathcal{U}$. We denote by $\mathcal{K}_\alpha(\beta)$ the subclass of $\mathcal{A}$ consisting of functions which are close-to-convex of order α and type β in $\mathcal{U}$. Also we write $\mathcal{K}_\alpha(0)\equiv\mathcal{K}_\alpha$.

Further, a function $f(z)$ in the class $\mathcal{A}$ is said to be a member of the class $\mathcal{R}(\alpha)$ if it satisfies

$$\mathrm{Re}\{f'(z)\}>\alpha \tag{1.3}$$

for some α $(0\leqq\alpha<1)$ and for all $z\in\mathcal{U}$.

For functions

$$f_j(z)=z+\sum_{n=2}^{\infty} a_{n,j} z^n \quad (j=1, 2) \tag{1.4}$$

belonging to the class $\mathcal{A}$, we denote by $f_1 * f_2(z)$ the convolution (or Hadamard product) of functions $f_1(z)$ and $f_2(z)$, that is

$$f_1 * f_2(z)=z+\sum_{n=2}^{\infty} a_{n,1} a_{n,2} z^n. \tag{1.5}$$

2. Convolution theorems. In order to derive our convolution theorems, we have to recall here the following lemmas due to Owa [2].

Lemma 1. *Let* $\phi(z)\in\mathcal{K}$ *and* $g(z)\in\mathcal{S}^*$. *If* $F(z)\in\mathcal{A}$ *and* $\mathrm{Re}\{F(z)\}>\alpha$ $(0\leqq\alpha<1; z\in\mathcal{U})$, *then*

$$\mathrm{Re}\left\{\frac{\phi * G(z)}{\phi * g(z)}\right\}>\alpha \quad (z\in\mathcal{U}), \tag{2.1}$$

where $G(z)=F(z)g(z)$.