34. *Unique Solvability of Nonlinear Fuchsian Equations*

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1. Introduction. Let $p\geq 2$ and $q\geq 0$ be integers, and let $x=(x_1, \cdots, x_p)$ and $y=(y_1, \cdots, y_q)$ be the variables in C^p and C^q, respectively. We denote by Z and N the set of integers and that of nonnegative integers, respectively. For a multi-index $\alpha=(\alpha_1, \cdots, \alpha_p)\in Z^p$, we set $x^\alpha=x_1^{\alpha_1}\cdots x_p^{\alpha_p}$, $|\alpha|=\alpha_1+\cdots+\alpha_p$.

Let $m\geq 1$. Then we shall prove the unique solvability of nonlinear Fuchsian equations

(1) $$a(x, y; D_x^\alpha D_y^\beta x^\gamma u; |\alpha|=|\gamma|\leq m, |\alpha|+|\beta|\leq m)=0,$$

where $a(x, y; z_{\alpha\beta\gamma})$ is a holomorphic function of x, y and $z=(z_{\alpha\beta\gamma})$. Because the study of the case $p=1$ is classical (cf. [1]), we are interested in the case $p\geq 2$. Madi [3] solved (1) under a so-called Poincare condition if $\alpha=\gamma$ and if (1) is linear. But, in the general case $\alpha\neq\gamma$, the definition of a Poincare condition is not clear. We also have a problem of a derivative loss which is caused by nonlinear terms in (1) such that $\beta\neq 0$.

We shall define a Poincare condition for (1) so that it extends the one in [3] in a natural way. Then we show the existence and uniqueness of solutions of (1) with an additional weak spectral condition (A. 3). A deeper connection between the generalized Poincare condition and the Hilbert factorization problem is also discussed.

The proof is done by a reduction to a system of equations on a scale of Banach spaces, which enables us to estimate the derivative loss of nonlinear terms.

2. Statement of results. We denote by $C_y\{\{x\}\}$ the set of all formal power series $\sum_{\alpha\in N^p} u_\alpha(y)x^\alpha$ where $u_\alpha(y)$ are analytic functions of y in some neighborhood of the origin independent of α. We denote by $C_y\{x\}$ the set of analytic functions of x and y at the origin. For a positive number $a\leq 1$, we define a ball B_a by $B_a=\{y\in C^q; |y_i|<a, i=1, \cdots, q\}$.

Let $A\subset\{\alpha\in Z^p; |\alpha|\geq 0\}$ and $B\subset N^q$ be finite sets. Let π be the projection onto $C_y\{\{x\}\}$;

(2) $$\pi x^\alpha u(x, y)=\sum_{\eta, \eta+\alpha\geq 0} u_\eta(y)x^{\eta+\alpha}, \quad u(x, y)=\sum_{\eta\geq 0} u_\eta(y)x^\eta\in C_y\{\{x\}\}.$$

We denote by $p_{\alpha\beta}(\partial)$ $(\alpha\in A, \beta\in B)$ multipliers of order $m_{\alpha\beta}$ given by

(3) $$p_{\alpha\beta}(\partial)v(x, y)=\sum_{\eta\geq 0} v_\eta(y)p_{\alpha\beta}(\eta)x^\eta, \quad v(x, y)=\sum_{\eta\geq 0} v_\eta(y)x^\eta\in C_y\{\{x\}\},$$

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