30. *Continuation of Bicharacteristics for Effectively Hyperbolic Operators*[†]

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1. Continuation for two pairs of bicharacteristics. We shall be concerned with regular—C^∞ or analytic—continuation of (null) bicharacteristics of an effectively hyperbolic differential or pseudo-differential operator. Given a hyperbolic principal symbol $p=p(\rho)=p(x,\xi)$ and an effectively hyperbolic characteristic point $\hat\rho=(\hat x,\hat\xi)$, we consider bicharacteristics $\rho=\rho(s)$ tending to $\hat\rho$ as $s\uparrow+\infty$ or $s\downarrow-\infty$. Our main result is then stated as follows (see also the final paragraph of this section):

Theorem 1. *There are exactly four such bicharacteristics. Two of them are incoming toward the reference point* $\hat\rho$ *with respect to the parameter* s*, and the other two are outgoing. Furthermore, each one of the incoming* (resp. *outgoing*) *bicharacteristics is naturally continued to the other one, and the resulting two curves are* C^∞ *regular near* $\rho=\hat\rho$ *as submanifolds of the cotangent bundle. These two curves are analytic near* $\rho=\hat\rho$ *whenever* $p=p(\rho)$ *is assumed to be analytic there.*

It has been known that the Cauchy problem for a hyperbolic operator is C^∞ well-posed for any lower order terms if and only if the principal symbol is effectively hyperbolic at every multiple (necessarily double) characteristic point. Effective hyperbolicity—introduced by Ivrii and Petkov—requires the existence of (necessarily two) non-zero real eigenvalues of the Hamilton map, i.e. the so-called fundamental matrix obtained by linearizing the Hamilton field of the principal symbol. (It turns out that such eigenvalues must be of the form $\pm\lambda$.) It may be natural to ask how this linear algebraic definition is reflected in the dynamical system of bicharacteristics near the reference point. The analysis proving Theorem 1 indicates that such a dynamical system can be regarded as a perturbation of that of the simplest model $p^0(y,\eta)=\eta_0^2-y_0^2$ in $T^*\boldsymbol{R}^{n+1}$, as far as the incoming and outgoing bicharacteristics are concerned. In this model, we may restrict ourselves to the (y_0,η_0)-plane, in which the origin is a *saddle point* of the Hamilton field of p^0. Here, $y=(y_0,y')$ and $\eta=(\eta_0,\eta')$.

The C^∞ continuation, as well as the existence, of exactly four such bicharacteristics was observed earlier by Iwasaki [4] using his difficult

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