18. On Algebroid Solutions of Some Binomial Differential Equations in the Complex Plane[†]

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1. **Introduction.** The purpose of this paper is to investigate algebroid solutions of some binomial differential equations in the complex plane with the aid of the Nevanlinna theory of meromorphic or algebroid functions.

Let $a_0, \cdots, a_p; b_0, \cdots, b_q$ be entire functions without common zero and put

$$P(z, w)=\sum_{j=0}^{p} a_j w^j, \quad Q(z, w)=\sum_{k=0}^{q} b_k w^k \quad (a_p \cdot b_q \neq 0).$$

We consider the differential equation (D. E.):

$$(1) \qquad (w')^n = P(z, w)/Q(z, w),$$

where n is an integer. We suppose that this equation is irreducible over the set of meromorphic functions in $|z|<\infty$ and that the D. E. (1) has a nonconstant ν-valued algebroid solution $w=w(z)$ in $|z|<\infty$.

Definition. We say that w is admissible when
$T(r, a_j/b_q)=o(T(r, w))$ $(0\leqq j\leqq p)$ and $T(r, b_k/b_q)=o(T(r, w))$ $(0\leqq k\leqq q-1)$ as $r\to\infty$, possibly outside a set of finite linear measure.

For example, when all a_j and b_k are polynomial, a transcendental algebroid solution of the D. E. (1) is admissible.

More than fifty years ago, K. Yosida ([11]) gave several results on algebroid solutions of the D. E. (1) when all a_j and b_k are polynomial. The followings are some of them.

Theorem A. *When all a_j and b_k are polynomial, w is of finite order and if w is transcendental,* $\max\{p, q+2n\}\leqq 2n\nu$.

There are generalizations of this theorem ([1], [3], [5], [8] etc.).

As a special case of a result of Y. He and X. Xiao ([3]), we have

Theorem B. *If w is admissible,* $p\leqq n+q+n\nu \limsup_{r\to\infty} \bar{N}(r, w)/T(r, w)$.

Recently, J. von Rieth ([6]) has studied the D. E. (1) based on K. Yosida's paper ([11]) and given some interesting results. The following is one of them.

Theorem C. *When all a_j and b_k are polynomial, if w is a transcendental solution with at most a finite number of poles, it must be $n+q\leqq p$.*

We note that in the case of Theorem C, it holds that $n+q=p$ according to Theorem B.

In this paper, we shall give some results on the solution of the D. E. (1)

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