107. On the Asymptotic Property of the Ordinary Differential Equation

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1. Introduction. In this paper we consider the asymptotic property of the zero solution of the ordinary differential equation

$$x'=f(t, x), \qquad f(t, 0)=0 \tag{1}$$

where x and f belong to the n-dimensional real space $\boldsymbol{R}^n$ with Euclidean norm $\|\cdot\|$, t is a real scalar and f is defined and continuously differentiable on $I\times\boldsymbol{R}^n$, $I=[0, \infty)$.

By Liapunov's direct method, Marachkoff proved the theorem for the asymptotic stability of the zero solution of (1) ([1]). He assumed that the derivative of the Liapunov's function with respect to (1): $V'_{(1)}(t, x)$ is negative definite. Some extensions of this condition were given by many authors ([2], [3] etc.). By using a second Liapunov's function $W(t, x)$, Matorosov extended Marachkoff's theorem in the case that $V'_{(1)}(t, x)$ is not negative definite ([4]).

The purpose of this paper is to extend some results related to an asymptotic property of the solution of (1) by means of a second Liapunov's function similar to that of Matorosov. Moreover we attempt to extend other conditions in Marachkoff's theorem. So we obtain some extensions of the above theorems.

2. Notations and definitions. We need some notations and definitions to state our results. If $x, y\in\boldsymbol{R}^n$, we denote the distance of x and y by $d(x, y)=\|x-y\|$. We denote by $E(V^*=0)$ the set $\{x\in\boldsymbol{R}^n: V^*(x)=0\}$, and denote by CIP the families of continuous strictly increasing, positive definite functions.

Definition 1. A function $\phi(t)$ is said to be integrally positive if $\int_S \phi(t)dt=+\infty$ holds on every set $S=\bigcup_{m=1}^{\infty}[\alpha_m, \beta_m]$ such that $\alpha_m<\beta_m<\alpha_{m+1}$, $\beta_m-\alpha_m\geq\delta>0$.

Definition 2. $W'_{(1)}(t, x)$ is said to be strictly not equal to zero in the set $E(V^*=0)$, if for any number α and A it is possible to find a number $r_1(\alpha, A)$ and a continuous functions $\xi(t)$ such that

$$\xi(t)>0, \quad \int_t^{\infty}\xi(s)ds=\infty \quad \text{for any } t$$

and in the set $\{(t, x): \alpha<\|x\|<A,\ d(x, E(V^*=0))<r_1,\ t\geq 0\}$

$$\|W'_{(1)}(t, x)\|\geq\xi(t)>0.$$

Definition 3. $W'_{(1)}(t, x)$ is said to be definitely not equal to zero in the