102. On an Extension of the James-Whitehead Theorem about Sphere Bundles over Spheres[†)]

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1. **Statement of results.** Let W be a handlebody obtained by gluing r, q-handles to a $(p+q+1)$-disk, and let $\mathcal{H}(p+q+1, r, q)$ be the set of such handlebodies. In this paper, I announce homotopy classification theorems of the boundaries of handlebodies of $\mathcal{H}(p+q+1, r, q)$ in the following two cases:

(1) $\qquad (p, q)=(n-1, n+1) \qquad (n\geq 4),$

(2) $\qquad (p, q)=(n-2, n+1) \qquad (n\geq 6).$

Such classifications are equivalent to those of simply connected closed m-manifolds M $(m=p+q)$ with $H_i(M)=0$ except for $i=0, p, q, m$ and with the tangent bundle which is trivial on its p-skeleton (this is satisfied if $p\equiv 3, 5, 6, 7 \bmod 8$). Henceforth, manifolds are connected, smooth, and oriented, and homotopy equivalences and diffeomorphisms are orientation preserving.

There exists an invariant system $(H\,;\phi, \alpha)$ which determines W up to diffeomorphism (cf. [4]). Here, $H=H_q(W)$, $\phi\colon H\times H\to Z_2=\pi_q(S^{p+1})$ is a symmetric bilinear form, and $\alpha\colon H\to\pi_{q-1}(SO_{p+1})$ is a quadratic form, which assigns, to each $x\in H\cong\pi_q(W)$, the characteristic element of the normal bundle of the imbedded q-sphere representing x. W is called of *type* 0 if $\phi=0$, of *type* II if $\phi(x, x)=0$ for any $x\in H$ and $\operatorname{rank}\phi=r$, and of *type* (0+II) if $\phi(x, x)=0$ for any $x\in H$ and $0<\operatorname{rank}\phi<r$. Note that ϕ is a homotopy invariant of ∂W by Proposition 1 of [2, II]. Our main purpose is to determine the necessary and sufficient condition for the boundaries of handlebodies to be homotopy equivalent using the invariant systems.

The following diagram is commutative up to sign:

$$\begin{tikzcd}
 & \pi_{q-1}(SO_p) \arrow[r, "S"] \arrow[d, "J"] & \pi_{q-1}(SO_{p+1}) \arrow[d, "J"] \\
\pi_q(S^p) \arrow[ur, "\partial"] \arrow[r, "{[\ ,\iota_p]=P}"'] & \pi_{m-1}(S^p) \arrow[r, "E"] & \pi_m(S^{p+1}),\ m=p+q.
\end{tikzcd}$$

Let $\lambda\colon S(\pi_{q-1}(SO_p))\to\pi_{m-1}(S^p)/\operatorname{Im}P$ be the homomorphism defined by $\lambda(S\xi)=\{J\xi\}$, which does not depend on the choice of ξ. Put $\theta=\eta_{n-1}$ if $(p, q)=(n-1, n+1)$ $(n\geq 4)$, and $\theta=\eta_{n-2}^2$ if $(p, q)=(n-2, n+1)$ $(n\geq 6)$. The inclusion map $i\colon S^p\to S^p\cup_\theta D^q$ induces the homomorphisms $i_*\colon\pi_{m-1}(S^p)\to\pi_{m-1}(S^p\cup_\theta D^q)$ and $\bar{i}_*\colon\pi_{m-1}(S^p)/\operatorname{Im}P\to\pi_{m-1}(S^p\cup_\theta D^q)/i_*(\operatorname{Im}P)$. We define $\bar{\lambda}\colon S(\pi_{q-1}(SO_p))\to\pi_{m-1}(S^p\cup_\theta D^q)/i_*(\operatorname{Im}P)$ by $\bar{\lambda}=\bar{i}_*\circ\lambda$.

Let W, W' be the handlebodies of $\mathcal{H}(p+q+1, r, q)$ with the invariant

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†) Dedicated to Professor Hirosi TODA on his 60th birthday.