99. *Tables of Ideal Class Groups of Real Quadratic Fields*

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§ 1. **Introduction.** A table of ideal class groups of imaginary quadratic fields $Q(\sqrt{-m})$ was given in [5] for $m<100,000$. In this note we shall give corresponding tables for ideal class groups in narrow sense and in wide sense for real quadratic fields $Q(\sqrt{m})$ for $m<100,000$. As in [5], we use the expression $(a, b, \cdots, c)$ to denote the type of finite abelian group which is the direct product of cyclic groups of order $a, b, \cdots, c$, $aZ\subset bZ\subset \cdots\subset cZ$. The ideal class groups in wide and narrow sense, the class numbers in wide and narrow sense, the fundamental unit, the 2-ranks of the ideal class groups in wide and narrow sense of $Q(\sqrt{m})$ and the number of rational primes ramified in $Q(\sqrt{m})$ are denoted by $C(m)$, $C'(m)$, $h(m)$, $h'(m)$, $\varepsilon(m)$, $r(m)$, $r'(m)$ and $t(m)$, (sometimes simply by C, C', h, h', ε, r, r', t) respectively. It is well known that $h'=h$ or $2h$ accoring as $N\varepsilon=-1$ or $+1$ and $r'=t-1$. We recall that a table of $h(m)$ and $N\varepsilon(m)$ is given in [4] for $m<100,000$. The method of our calculation is based on [2] Chapter 5. It was done by micro VAX-II and the computer time for making these tables was about 40 hours.

§ 2. **Ideal class groups in narrow sense.** Our Table I gives the types $(a, b, \cdots, c)$ of $C'(m)$ for all $m<100,000$ except in the following two cases:

(1) $C'(m)$ is cyclic.

(2) $C'(m)$ is of the type $(2a', 2, \cdots, 2)$ and $t>2$.

Thus when m is not found in Table I, and $t=1$ or 2, then $C'(m)$ is cyclic, and when $t>2$ then $C'(m)$ is of the type $(2a', 2, \cdots, 2)$ with $a'=h'/2^{t-1}$.

§ 3. **Ideal class groups in wide sense.** If $N\varepsilon(m)=-1$, it is well-known that $C'(m)$ and $C(m)$ are of the same type. We have furthermore the following theorem.

Theorem. *Let $R(m)$ be the set of rational primes ramified in $Q(\sqrt{m})$ (i.e. the set of prime divisors of the discriminant of $Q(\sqrt{m})$).*

(1) *If $R(m)$ contains a prime $\equiv 3 \pmod 4$, then*

$$r(m)=r'(m)-1=t-2.$$

(2) *Otherwise* $r(m)=r'(m)=t-1$.

The proof of this theorem is implicitly contained in [1] or in [3], but this explicit formulation was communicated to us by Prof. Iwasawa. We add here a short proof for convenience.

Proof. In case (1), the norm of the fundamental unit is 1 and there is no number $\theta\in Q(\sqrt{m})$ satisfying $N(\theta)=-1$. So $r=t-2$ ([3] p. 257).

In case (2), we can conclude from calculation of the Hilbert Symbol that