88. *The Sylvester's Law of Inertia for Jordan Algebras*

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The purpose of this note is to present some results on the orbit structure of a compact (=formally real) simple Jordan algebras under the action of the identity component of its structure group. In view of the classification of compact simple Jordan algebras, Theorem 1 is viewed as a natural generalization of the Sylvester's law of inertia for real symmetric or complex Hermitian matrices. We shall use terminologies and well-known facts in the theory of Jordan algebras without giving explanations (see, for instance, Jacobson [2] and Braun-Koecher [1]).

1. Let $\mathfrak{A}$ be a compact simple Jordan algebra of degree r, and let $G(\mathfrak{A})$ be the structure group of $\mathfrak{A}$. Let $G^0(\mathfrak{A})$ denote the identity component of $G(\mathfrak{A})$. Let $a \in \mathfrak{A}$ and let

$$m_a(\lambda)=\lambda^r-\sigma_1(a)\lambda^{r-1}+\cdots+(-1)^r\sigma_r(a) \tag{1}$$

be the generic minimum polynomial of a (for details, see [2]). Note that each $\sigma_i(a)$ is a homogeneous polynomial of degree i in the components of a. If we denote the minimum polynomial of the element a by $\mu_a(\lambda)$, then each irreducible factor of $m_a(\lambda)$ is a factor of $\mu_a(\lambda)$ ([2]). The polynomial equation $\mu_a(\lambda)=0$ has only real roots, since $\mathfrak{A}$ is compact ([1]). Therefore the equation $m_a(\lambda)=0$ also has only real roots. By the *signature* of an element $a \in \mathfrak{A}$ (denoted by sgn (a)), we mean the pair of the integers (p, q) such that p and q are numbers of positive and negative roots of the equation $m_a(\lambda)=0$, respectively. Here the number of a root should be counted by including its multiplicity. Let $\mathfrak{A}_{p,q}$ denote the set of elements $a \in \mathfrak{A}$ with sgn $(a)=(p, q)$. Then we have

$$\mathfrak{A}=\coprod_{p+q\leqslant r}\mathfrak{A}_{p,q}. \tag{2}$$

Now let e be the unit element of $\mathfrak{A}$. Since $\mathfrak{A}$ is of degree r, one can choose a system of primitive orthogonal idempotents $\{e_1, \cdots, e_r\}$ of $\mathfrak{A}$ such that $\sum_{i=1}^r e_i=e$. Such systems are conjugate to each other under the automorphism group Aut $\mathfrak{A}$ of $\mathfrak{A}$. We choose and fix such a system $\{e_1, \cdots, e_r\}$ and put

$$o_{p,q}=\sum_{i=1}^{p}e_i-\sum_{j=p+1}^{p+q}e_j, \quad p, q\geqslant 0, \quad p+q\leqslant r; \tag{3}$$

here we are adopting the convention that the first and the second terms of the right hand side of (3) should be zero, provided that $p=0$ and $q=0$, respectively.

Theorem 1. *Let $\mathfrak{A}$ be a compact simple Jordan algebra of degree r. Then the decomposition* (2) *is the G^0 $(\mathfrak{A})$-orbit decomposition of $\mathfrak{A}$. More*