10. Lifting of Local Subdifferentiations and Elliptic Boundary Value Problems on Symmetric Domains. II

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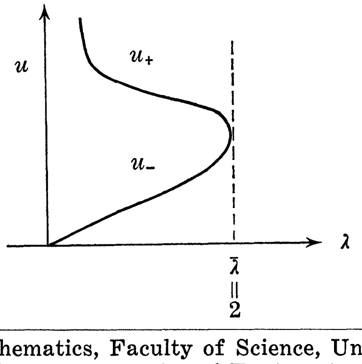
Our purpose is to study nonlinear eigenvalue problem

(1) $\quad -\Delta u=\lambda e^u \quad (\text{in } \Omega), \quad u=0 \quad (\text{on } \partial\Omega)$

for $\lambda>0$ on $\Omega=\{x \mid a<|x|<1\}\subset \boldsymbol{R}^2$, where $0<a<1$. From variational method, we shall show the existence of multiple non-radial solutions for (1). Namely, we seek the solutions by lifting of local subdifferentiations developed in [6], and then separate critical values by Steiner's symmetrization according to the argument by Kawohl [3]. Meanwhile we shall make use of radial solutions for (1) on the ball. Thus, our plan is; i. Description of the solutions for (1) on $\Omega_0=\{|x|<1\}$, ii. Description of radial solutions for (1) on $\Omega_a=\{a<|x|<1\}$ $(0<a<1)$ and iii. Existence of non-radial solutions for (1) on Ω_a.

We note that the equation $-\Delta u=\lambda e^u$ (in Ω) has an integral (Liouville [4]). Thus it is equivalent to $(\lambda/8)^{1/2}e^{u/2}=\rho(F)=|F'|/(1+|F|^2)$, where F is a meromorphic function on Ω such that $\rho(F)>0$. Therefore, (1) is nothing but to find F such that $\rho(F)|_{\partial\Omega}=(\lambda/8)^{1/2}$.

Solutions of (1) *for* $\Omega_0=\{|x|<1\}\subset \boldsymbol{R}^2$: Every solution $u=u(x)$ of (1) is positive so that is radial in this case (Gidas-Ni-Nirenberg [2]). Hence the result of Gel'fand [1] supplies a complete diagram of the solutions of (1). In terms of the Liouville integral given above, they are given through $F(z)=Cz$ with a $C>0$ satisfying $\rho(F)|_{\partial\Omega}=C/(1+C^2)=(\lambda/8)^{1/2}$. Hence for $\lambda>2$ (1) has no solution. According to $\lambda=2$ and $0<\lambda<2$, (1) has exactly one and two solutions $u=u_\pm$. They are described through the parameter $\kappa=1/C^2$, which is given as $\kappa^{1/2}=\kappa_\pm^{1/2}=(2/\lambda)^{1/2}(1\mp\sqrt{1-\lambda/2})$ for $0<\lambda\leq 2$. That is, $(\lambda/8)^{1/2}e^{u_\pm/2}=\kappa_\pm^{1/2}/(|x|^2+\kappa_\pm)$. See the figure given below.



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