75. *Structural Operators for Linear Delay-differential Equations in Hilbert Space*

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Let H and V be complex Hilbert spaces such that V is a dense subspace of H and the inclusion mapping of V into H is continuous. The norms of H and V are denoted by $|\ |$ and $\|\ \|$ respectively. Identifying H with its antidual we may write $V \subset H \subset V^*$. We use the notation $(\ ,\ )$ to denote both the innerproduct of H and the pairing between V^* and V. For a couple of Hilbert spaces X and Y the notation $B(X, Y)$ denotes the totality of bounded linear mappings of X into Y, and $B(X)=B(X, X)$.

Let $a(u, v)$ be a sesquilinear form defined on $V \times V$. Suppose that there exist positive constants C and c such that

$$|a(u, v)| \leqq C \|u\| \|v\|, \qquad \operatorname{Re} a(u, u) \geqq c \|u\|^2$$

for any $u, v \in V$. Let $-A_0 \in B(V, V^*)$ be the operator associated with this sesquilinear form: $(-A_0 u, v)=a(u, v)$, $u, v \in V$. The realization of A_0 in H which is the restriction of A_0 to $D(A_0)=\{u \in V : A_0 u \in H\}$ is also denoted by the same letter A_0. It is known that A_0 generates an analytic semigroup in both H and V^*.

Let A_i, $i=1,2$, be operators in $B(V, V^*)$. Then, $A_i A_0^{-1} \in B(V^*)$ for $i=1,2$. We assume that these two operators map H to itself and $A_i A_0^{-1} \in B(H)$, $i=1,2$. We assume also that $A_i^*(A_0^*)^{-1} \in B(H)$, $i=1,2$, where $A_0^*, A_i^* \in B(V, V^*)$ are the adjoint operators of A_0, A_i.

Let $a(s)$ be a real valued Hölder continuous function in the interval $[-h, 0]$, where h is some positive number. We consider the following delay-differential equation

$$du(t)/dt = A_0 u(t) + A_1 u(t-h) + \int_{-h}^0 a(s) A_2 u(t+s) ds \tag{1}$$

which is considered as an equation in both H and V^*. According to [3] the fundamental solution $W(t)$ of (1) can be constructed.

It is easily seen that the space

$$\left\{ f \in V^* : \int_0^\infty \|A_0 \exp(tA_0) f\|_*^2 \, dt < \infty \right\}$$

coincides with H, where $\|\ \|_*$ is the norm of V^*. Hence, in view of [1] the semigroup $S(t)$ in $Z = H \times L^2(-h, 0; V)$ is defined by

$$S(t)g = (u(t; g), u(t+\cdot\,; g)), \qquad g=(g^0, g^1) \in Z$$

where $u(t; g)$ is the mild solution of (1) (cf. [2]) satisfying the initial condition

$$u(0; g) = g^0, \quad u(s; g) = g^1(s), \quad -h \leqq s < 0.$$