8. *Complex Analytic Compactifications of C^3*

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Let X be an n-dimensional connected compact complex manifold and Y an analytic subset of X. We call the pair (X, Y) a complex analytic compactification of C^n if $X-Y$ is biholomorphic to C^n. For $n=1$, it is easy to see that $(X, Y)\cong(P^1, \infty)$. For $n=2$, Remmert-Van de Ven [6] proved that $(X, Y)\cong(P^2, P^1)$ if Y is irreducible, where $Y=P^1$ is a line in P^2.

In this note, we will study the case in which $n=3$. Our main result is the following

Theorem. *Let (X, Y) be a complex analytic compactification of C^3. Assume that Y is normal. Then $(X, Y)\cong(P^3, P^2)$, (Q^3, Q_0^2) or (V_5, H_5), where $Q^3 \hookrightarrow P^4$ is a smooth quadric hypersurface, Q_0^2 is a quadric cone, V_5 is a Fano 3-fold of degree 5 in P^6 and H_5 is a hyperplane section of V_5 which has a singularity of A_4-type.*

Remark. These pairs (P^3, P^2), (Q^3, Q_0^2), (V_5, H_5) really exist.

Sketch of Proof. Let (X, Y) be as in Theorem. The normality of Y implies the projectivity of X (cf. [1]). Then X is a Fano 3-fold with $b_2(X)=1$. Let $r(0<r\leqq 4)$ be the index of X. In the case of $r\geqq 2$, we have proved in [2] the following results:

(i) $r=4\Rightarrow(X, Y)\cong(P^3, P^2)$
(ii) $r=3\Rightarrow(X, Y)\cong(Q^3, Q_0^2)$
(iii) $r=2\Rightarrow(X, Y)\cong(V_5, H_5)$.

Therefore we have only to show that the case of $r=1$ can not occur.

Assume that such a (X, Y) exists in the case of $r=1$. Then, by the classification of Fano 3-folds due to Iskovskih [3] and the detailed analysis of the singularities of the boundary Y, we find that $(X, Y)\cong(V_{22}, H_{22})$, where $X=V_{22}$ is a Fano 3-fold of degree 22 in P^{13} and $Y=H_{22}$ is a hyperplane section of V_{22} which has a minimally elliptic singularity x of type $A_{3,**,o}+D_{5,*,o}$ in the terminology of Laufer [6].

Next, let us consider the triple projection of V_5 from the singularity x of Y. Then we have the diagram below:

$$\begin{array}{ccc} V'_{22} & \xrightarrow{\rho} & W \\ \sigma\downarrow & & \downarrow\pi \\ V_{22} & \overset{\varphi}{\dashrightarrow} & Q \end{array}$$

where

(1) $\sigma: V'_{22}\to V_{22}$ is the blowing up of V_{22} at the point $x\in Y\subset X$.
(2) W is a smooth 3-fold and $\pi: W\to Q$ is a conic bundle over a quadric