69. *Local Deformation of Pencil of Curves of Genus Two*

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§ 1. **Introduction.** Let S be a compact complex surface which admits a surjective holomorphic map $g: S \to \Delta$ onto a compact Riemann surface Δ. We suppose that the general fibres are smooth curves of genus 2. Then S is birationally equivalent to a branched double covering S' over a $\boldsymbol{P}^1$-bundle W over Δ whose branch locus B intersects a general $\boldsymbol{P}^1$ at 6 points. Though there are infinitely many choices of W, we can choose one, by applying elementary transformations to W, such that the branch locus B is, in some sense, canonical. After this is done, the singular fibres of g are classified into six types (0), (I_k), (II_k), (III_k), (IV_k) and (V) (see [4]). Recall that the singular fibres of type (0) are obtained by resolving only rational double points on the singular model S', and that the most general singular fibres of type (I_1) are composed of two elliptic curves with self-intersection number -1 which intersect transversally at one point (they will be called general (I_1) type).

In this paper we study deformations of surfaces with such fibration, but only locally at each singular fibre. More precisely, let $g^{-1}(P)$, $P \in \Delta$ be a singular fibre of S and let U be a small neighborhood of P and $X = g^{-1}(U)$. Then we shall prove the following theorem.

Theorem. *Assume $g^{-1}(P)$ is a singular fibre of type* (T) *other than type* (0). *Then there exists a family $\{X_t\}_{t \in M}$ of deformations of $X = X_0$, $0 \in M$ such that*

i) *each X_t admits a holomorphic map $g_t: X_t \to U$ whose general fibre is of genus* 2, *and g_t depends holomorphically on t,*

ii) *for general $t \in M$, $g_t: X_t \to U$ has only singular fibres of general* (I_1) *type and type* (0),

iii) *the number $\delta(\mathrm{T})$ of these singular fibres of general* (I_1) *type in g_t is given by*

$$\delta(\mathrm{I}_k) = \delta(\mathrm{III}_k) = 2k-1, \quad \delta(\mathrm{II}_k) = \delta(\mathrm{IV}_k) = 2k, \quad \delta(\mathrm{V}) = 1.$$

This theorem states that each singular fibre of type (T) is, in some sense, "equivalent" to $\delta(\mathrm{T})$ singular fibres of general (I_1) type modulo those of type (0). Recall that the value $\delta(\mathrm{T})$ equals the contribution of the singular fibre of type (T) to the difference $c_1^2 - (2\chi + 6(\pi - 1))$, where $\chi = \chi(\mathcal{O}_S)$, π is the genus of Δ and the Chern number c_1^2 is the value for relatively minimal S [4, Theorem 3].

The result is related to the construction of a family of deformations of elliptic double points which admits simultaneous resolution. To conclude