55. *Initial Boundary Value Problem for the Equations of Ideal Magneto-Hydro-Dynamics with Perfectly Conducting Wall Condition*

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1. In this paper we consider the initial boundary value problem for the equations of ideal MHD that describe the motion of an ideal plasma filling an open subset of $\boldsymbol{R}^3$, surrounded by a rigid and perfectly conducting wall. (See [1].) Our problem is to solve

$$(1)_a \qquad \rho_p(\partial_t+(u\cdot\nabla))p+\rho\nabla\cdot u=0$$

$$(1)_b \qquad \rho(\partial_t+(u\cdot\nabla))u+\nabla p+\mu H\times(\nabla\times H)=0$$

$$(1)_c \qquad \partial_t H-\nabla\times(u\times H)=0 \qquad \text{in } [0, T]\times\Omega,$$

$$(1)_d \qquad (\partial_t+(u\cdot\nabla))S=0$$

$$(1)_e \qquad \nabla\cdot H=0$$

$$(2) \qquad (p, u, H, S)|_{t=0}=(p_0, u_0, H_0, S_0)\equiv U_0 \qquad \text{in } \Omega,$$

$$(3) \qquad u\cdot n=0,\ H\cdot n=0 \qquad \text{on } [0, T]\times\Gamma.$$

Here Ω is a bounded or unbounded domain in $\boldsymbol{R}^3$ with a smooth and compact boundary Γ, or a half space $\boldsymbol{R}^3_+$; the pressure p, the velocity $u=(u^1, u^2, u^3)$, the magnetic field $H=(H^1, H^2, H^3)$, and the entropy S are the unknown functions of t and x; the density ρ is determined by the equation of state $\rho=\rho(p, S)$; $\rho>0$ and $\rho_p=\partial\rho/\partial p>0$ for $p>0$; the magnetic permeability μ is assumed to be constant; we write $\partial_t=\partial/\partial t$, $\partial_i=\partial/\partial x_i$, $\nabla=(\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ and use the conventional notations in vector analysis; $n=n(x)=(n_1, n_2, n_3)$ denotes the unit outward normal at $x\in\Gamma$.

2. We set $U={}^t(p, u, H, S)$ and rewrite the system $(1)_{a\text{-}d}$ in the symmetric form

$$(4) \qquad A_0(U)\partial_t U+\sum_{i=1}^{3} A_i(U)\partial_i U=0.$$

In order to solve the problem by iteration, we consider the linearization of (4) around an arbitrary function $U'={}^t(p', u', H', S')$ near the initial data, satisfying $u'\cdot n=0$ and $H'\cdot n=0$ on Γ. The linearized equation forms a symmetric hyperbolic system with singular boundary matrix. In fact, the boundary matrix has constant rank 2 on Γ. We define $X^m(T, \Omega)$ to be the space of functions $U(t, x)$ taking values in $\boldsymbol{R}^8$ and satisfying the following property: Let $\beta\geq 0$ be an integer and let $\Lambda_1, \cdots, \Lambda_\beta$ be an arbitrary β-tuple of smooth and bounded vector fields tangential to Γ, namely, let $\langle\Lambda_i(x), n(x)\rangle=0$ for $x\in\Gamma$, $i=1, \cdots, \beta$. Then $\partial_t^\alpha\Lambda_1\cdots\Lambda_\beta\partial_n^k U(t, x)\in L^\infty(0, T; L^2(\Omega))$ for $\alpha+\beta\leq m-2k$, $k=0, 1, \cdots, [m/2]$. Here ∂_n denotes the partial differentia-

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