51. An Inequality of Chern Numbers of Bogomolov Type for Minimal Varieties

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1. Introduction. Recently Mori proved the existence of minimal models for projective algebraic 3-folds ([5]) and it is expected that minimal models for projective manifolds exist in all dimension. Hence it is important to study minimal algebraic varieties.

In [6], the author proved semistability of tangent bundles of smooth minimal algebraic varieties with respect to the canonical polarization and an inequality of Chern numbers of Miyaoka-Yau type for them. In [1] Enoki generalized the author's result about semistability to the case of minimal Kähler spaces. Actually he proved semistability of tangent sheaves with respect to the canonical (weak) polarization for minimal Kähler spaces ([1, Theorem 1.1]).

Their methods are a little bit different because there are no ample divisors on Kähler spaces. Actually the author considered not only a perturbation of the canonical polarization but also a perturbation of the tangent bundle itself in [6] while Enoki used only a perturbation of the canonical polarization in [1]. This explains why Enoki's method does not yield an inequality of Chern numbers.

The purpose of this short note is to show that we can easily get an inequality of Chern numbers of Bogomolov type for minimal varieties just by combining the both methods.

Theorem 1. *Let X be a minimal algebraic variety of dimension $n(\geq 3)$ over $\mathbf{C}$ such that codimSing $X>2$. Then the inequality*

$$(-1)^n c_1^n(X) \leq (-1)^n \frac{2n}{n-1} c_1^{n-2}(X) c_2(X)$$

holds.

We note that this inequality is a corollary of the result in [4] in the case of $n=3$. In the case that K_X is ample, we may deduce the inequality by using [1, Theorem 1.1], [3] and [2]. Unfortunately this inequality does not seem to be sharp. This inequality can be proved by constructing a Kähler metric with a good control of the Ricci tensor. To get a sharp inequality (i.e. an inequality of Miyaoka-Yau type), it seems to be necessary to construct some (singular) Kähler-Einstein metric on X with a good control of the full curvature tensor.

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