45. A Mathematical Theory of Randomized Computation. II

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(Communicated by Shokichi IYANAGA, M. J. A., May 12, 1988)

Keeping up the discussion of randomized domains begun in our first note [6], we shall now establish their characterization.

Abstracting the Fatou lemma, the B. Levi theorem, and the Lebesgue dominated convergence theorem from the Lebesgue integration, we define the norm topology on a *BL* V as follows. We shall write $\|D\| := \{\|x\| \mid x \in D\}$ for $\forall D \subset V$:

(10) (i) The norm topology on V is *Fatou* if the norm $\|\ \|$ is a Fatou norm. A *Fatou* norm on V is a lattice norm $\|\ \|$ s.t. $D \uparrow d$ in $V^+ \Rightarrow \|d\| = \sup \|D\|$. In this case V is said to *have the Fatou property*. (ii) The norm topology on V is *Levi* if the norm $\|\ \|$ is a Levi norm. A *Levi* norm on V is a lattice norm $\|\ \|$ s.t. $D \uparrow$ in V^+ and $\sup \|D\| < \infty \Rightarrow \exists d = \sup D \in V$. In this case V is said to *have the Levi property*. (iii) The norm topology on V is *Lebesgue* if the norm is order continuous norm (or Lebesgue norm). An *order continuous* norm (or *Lebesgue* norm) on V is a lattice norm $\|\ \|$ s.t. $D \downarrow 0 \Rightarrow \|D\| \downarrow 0$. In this case V is said to *have the Lebesgue property* (or, V is *order continuous*). In (i), (ii), and (iii), if D is any countable sequence then we prefix σ- to the terms Fatou, Levi, and Lebesgue, respectively.

Now we shall identify the order topology on a randomized domain with the Scott topology: In fact, a "well defined" value x input to a randomized program, is understood to occur in the program with probability 1, thus denoting the point mass 1_x, while the undefined value $\perp$ is understood to occur in the program with probability 0, thus denoting zero probability measure 0. In higher types, a Scott continuous function corresponds to a positive order continuous operator. Moreover, it must be intuitively true that any data should be "defined with respect to the degree of definition" prior to be "measured by some metric system". So if every directed set Z has a supremum z in the Scott topology, then the metrized set $\|Z\|$ must converge to $\|z\|$ in any metric $\|\ \|$.

This observation leads us to the idea of postulating the order continuity of the norm on the *BL* in which a randomized domain lies. So with Vulich [5] we give the following definitions:

(11) (i) A *KB-space* is a *BL* whose norm topology is Levi and Lebesgue. (ii) A *KB*-space V is *algebraic* if the positive cone V^+ is an algebraic ccp