5. *Characterization of the Eigenfunctions in the Singularly Perturbed Domain. II*

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In this paper, we give some elaborate estimates concerning the eigenfunction which behaves singularly when the domain is singularly perturbed. J.T. Beale [1] has characterized the set of scattering frequencies (i.e. the square root of the spectrum) of the exterior domain of a bounded obstacle with a partially open cavity when the channel to the cavity is very narrow. In our previous works [2] and [3], we have dealt with a Dumbell type domain; $\Omega(\zeta)=D_1\cup D_2\cup Q(\zeta)$ where $Q(\zeta)$ approaches a line segment as $\zeta\to 0$ (which is a similar domain perturbation to that of J. T. Beale) and we have characterized the eigenfunctions of the operator $-\Delta$ in the case of the Neumann boundary condition. Roughly speaking, the complete system of the eigenvalues $\{\mu_k(\zeta)\}_{k=1}^{\infty}$ and the eigenfunctions $\{\Phi_{k,\zeta}\}_{k=1}^{\infty}$ orthonormalized in $L^2(\Omega(\zeta))$ are separated as follows,

$$\{\mu_k(\zeta)\}_{k=1}^{\infty}=\{\omega_k(\zeta)\}_{k=1}^{\infty}\cup\{\lambda_k(\zeta)\}_{k=1}^{\infty}$$
$$\{\Phi_{k,\zeta}\}_{k=1}^{\infty}=\{\phi_{k,\zeta}\}_{k=1}^{\infty}\cup\{\psi_{k,\zeta}\}_{k=1}^{\infty}$$

where

$$\lim_{\zeta\to 0}\|\phi_{k,\zeta}\|_{L^2(Q(\zeta))}=0,\qquad \lim_{\zeta\to 0}\|\psi_{k,\zeta}\|_{L^2(D_1\cup D_2)}=0,$$
$$\limsup_{\zeta\to 0}\|\phi_{k,\zeta}\|_{L^\infty(\Omega(\zeta))}<+\infty,\quad \lim_{\zeta\to 0}\|\psi_{k,\zeta}\|_{L^\infty(\Omega(\zeta))}=+\infty.$$

More precisely, $\phi_{k,\zeta}$ approaches the k-th eigenfunction on $D_1\cup D_2$ uniformly and $\psi_{k,\zeta}$ approaches the k-th eigenfunction

$$\frac{1}{d_{n-1}^{1/2}\zeta^{(n-1)/2}}\sin\frac{1}{2}k\pi(x_1+1)$$

of $-d^2/dx_1^2$ on the line segment $L=\bigcap_{\zeta>0}\overline{Q(\zeta)}$ with the Dirichlet boundary condition on the endpoints of L in some sense. The asymptotic behavior of $\phi_{k,\zeta}$ when $\zeta\to 0$ has been obtained globally in $\Omega(\zeta)$ in [2]. In this paper we obtain the exact decay estimate of $\psi_{k,\zeta}$ in $D_1\cup D_2$ when $\zeta\to 0$. The estimates or methods obtained are very useful when we deal with a construction of the solutions of some semilinear elliptic equation on the singularly perturbed domain.

§1. Formulation. We specify the singularly perturbed domain $\Omega(\zeta)$ in $\boldsymbol{R}^n$ in the following form,

$$\Omega(\zeta)=D_1\cup D_2\cup Q(\zeta)$$

where D_i $(i=1, 2)$ and $Q(\zeta)$ are defined in the following conditions where $x'=(x_2, x_3, \cdots, x_n)\in\boldsymbol{R}^{n-1}$.

(A.1) D_1 and D_2 are bounded domains in $\boldsymbol{R}^n$ (mutually disjoint) with