20. The Moduli Space of Hermite-Einstein Bundles on a Compact Kähler Manifold

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In this note we shall give the construction of the moduli space $\mathcal{M}_{\mathrm{HE}}$ of holomorphic irreducible Hermite-Einstein vector bundles on a compact manifold X. This space is introduced as a finite dimensional real analytic subspace of the $\boldsymbol{R}$-Banach analytic manifold of isomorphism classes of irreducible $U(r)$-connections on a hermite vector bundle $E\to X$. The map, which assigns the corresponding semi-connection to a Hermite-Einstein connection descends to a real analytic injective local isomorphism to the complex analytic (not necessarily Hausdorff) moduli space of simple holomorphic vector bundles on X. In particular $\mathcal{M}_{\mathrm{HE}}$ is a Hausdorff, complex space.

A construction of the regular part of $\mathcal{M}_{\mathrm{HE}}$, including differential geometric investigations, was achieved by N. Koiso [5]. Independently M. Lübke and C. Okonek worked on this subject. Our method is a direct generalization of Ito [4].

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Holomorphic vector bundles, whose $\boldsymbol{C}$-endomorphisms consist just of homotheties (constant multiples of the identity), are called *simple*. An immediate consequence of the principles of deformation theory is:

Theorem 1. *Let X be a compact complex manifold, then the set $\mathcal{M}_s$ of isomorphism classes of simple holomorphic vector bundles carries the natural structure of a (not necessarily Hausdorff) complex space.*

The proof follows from a general argument of [3], [6]: If S is a complex space, $s_0 \in S$ a point and $V\to X\times S$ a family of simple holomorphic vector bundles, then all automorphisms of $V_{s_0}=V|X\times\{s_0\}$ can be extended to isomorphisms of the families over a neighborhood of s_0; a fact, which implies the existence of universal deformations; and if $V\to X\times S$ and $W\to X\times R$ are universal families, such that V_{s_0}, $s_0\in S$ and W_{r_0}, $r_0\in R$ are isomorphic, then there exists a uniquely determined isomorphism of neighborhoods of r_0 and s_0 resp., which can be lifted to the families of bundles.

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