19. The Dimension of the Space of Relatively Invariant Hyperfunctions on Regular Prehomogeneous Vector Spaces

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(Communicated by Kôsaku YOSIDA, M. J. A., March 12, 1987)

1. Introduction. Let G_C be a connected complex algebraic group which is a linear algebraic subgroup of $GL(V_C)$ where V_C is a complex finite dimensional vector space. We suppose that there exists an irreducible non-degenerate polynomial $P(x)$ (i.e., $\det((\partial^2 P(x))/(\partial x_i \partial x_j))$ does not identically vanish) such that $P(x)$ is relatively invariant with respect to G_C and $V_C - S_C$ is the unique open orbit where $S_C := \{x \in V_C;\, P(x)=0\}$. Then we have $P(g\cdot x)=\chi(g)P(x)$ for all $g \in G_C$ with a character $\chi(g)$. This means that (G_C, V_C) is a regular prehomogeneous vector space defined over the complex field C. Let V_R be a real form of V_C such that $G_R := GL(V_R)\cap G_C$ is a real form of G_C. We denote by G_R^+ the connected component of G_R containing the identity element. For a hyperfunction $f(x)$ on V_R, we say that $f(x)$ is $|\chi|^\lambda$-*invariant* ($\lambda \in C$) if it satisfies the equation $f(g\cdot x)=|\chi(g)|^\lambda f(x)$ for all $g \in G_R^+$. We denote by $\mathcal{B}^{G_R^+}(|\chi|^\lambda)$ the space of $|\chi|^\lambda$-invariant hyperfunctions on V_R. The purpose of this note is to report that, for almost all reduced regular irreducible prehomogeneous vector spaces (G_C, V_C), we can prove that the dimension of $\mathcal{B}^{G_R^+}(|\chi|^\lambda)$ coincides with $l :=$ the number of connected components of $V_R-(V_R\cap S_C)$. Moreover it is proved that they are written as a linear combination of the complex powers of $P(x)$ supported on the closures of connected components of $V_R-(V_R\cap S_R)$. It has been proved that the dimension of $\mathcal{B}^{G_R^+}(|\chi|^\lambda)$ is greater than l in a very general setting. See Muro [3], Oshima-Sekiguchi [7] and Ricci-Stein [8]. The crucial point is the upper estimate of the dimension of $\mathcal{B}^{G_R^+}(|\chi|^\lambda)$.

These results are obtained as an application of microlocal analysis. In particular, the author has already proved the same theorem for almost all real forms of regular prehomogeneous vector spaces of commutative parabolic type (defined in Muller-Rubenthaler-Schiffmann [6]) in [3]. What he wants to stress in this note is that the same method employed there works well for a wider class of regular prehomogeneous vector spaces.

2. Problem. The real locus of the open orbit $V_R-(V_R\cap S_C)$ decomposes into a finite number of connected components. Each connected component is an open G_R^+-orbit. We denote by $V_1\cup\cdots\cup V_l$ its connected component decomposition. We define a tempered distribution $|P(x)|_i^s$ ($s \in C$ and $i=1,\cdots,l$) in the following way: When the real part of s is sufficiently large, $|P(x)|_i^s$ is defined to be a continuous function which is $|P(x)|^s$ on $x \in V_i$