16. *Applications of Spreading Models to an Equivalence of Summabilities and Growth Rate of Cesàro Means*

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0. Introduction. In this paper, we present two applications of Brunel-Sucheston spreading models. One application is to estimate, from above, the growth rate of Cesàro means and the other one is to discuss an equivalence between regular methods of summability. The complete proofs of our results and related ones will appear elsewhere.

Throughout this paper, X denotes a Banach space, N denotes the set of all positive integers, and S_0 denotes the vector space of finite scalar sequences with the canonical unit vector basis $\{e_n\}_n$.

1. Brunel-Sucheston spreading model. We start by explaining the concept of the Brunel-Sucheston spreading model. Let $\{x_n\}_n$ be a bounded sequence with no norm Cauchy subsequence in a Banach space X. Suppose that the limit

$$\lim_{\substack{m\to\infty \\ m\leqq n_1<\cdots<n_k}} \left\|\sum_{i=1}^{k} a_i x_{n_i}\right\|$$

exists for all $(a_i)_{i=1}^k$ in S_0. We shall call such a sequence $\{x_n\}_n$ a *BS-sequence* (named after Brunel and Sucheston). Then we can define the nonnegative function Ψ on S_0 by

$$\Psi((a_i)_{i=1}^k) := \lim_{\substack{m\to\infty \\ m\leqq n_1<\cdots<n_k}} \left\|\sum_{i=1}^{k} a_i x_{n_i}\right\|.$$

It is known that Ψ defines a norm on S_0 (see [3, p. 296]), hence we shall write $\|\sum_{i=1}^k a_i e_i\|$ in place of $\Psi((a_i)_{i=1}^k)$ for each $(a_i)_{i=1}^k$ in S_0. Let E be the completion of $[S_0, \|\cdot\|]$. We say that $[E, \{e_n\}_n]$ is the *spreading model* of $\{x_n\}_n$. In [3], Brunel and Sucheston proved that every bounded sequence in any Banach space with no norm Cauchy subsequence has a subsequence which is a *BS*-sequence. Then $\{x_n\}_n$ and its spreading model $[E, \{e_n\}_n]$ have the following properties (Spreading Model):

(1) $$\left\|\sum_{i\in A_1} a_i(e_{2i-1}-e_{2i})\right\| \leqq \left\|\sum_{i\in A_2} a_i(e_{2i-1}-e_{2i})\right\|$$

for each finite subsets A_1, A_2 of N with $A_1 \subset A_2$ and $(a_i)_i$ in S_0.

(2) $$\lim_{\substack{m\to\infty \\ m\leqq n_1<\cdots<n_k}} \left\|\sum_{i=1}^{k} a_i x_{n_i}\right\| = \left\|\sum_{i=1}^{k} a_i e_i\right\|$$

for every vector $(a_i)_{i=1}^k$ in S_0.

(3) For any $\varepsilon>0$ and k in N there exists an $L(\varepsilon, k)$ in N so that for every $(a_i)_{i=1}^k$ in S_0 and n_i in N $(i=1,2,\cdots,k)$ with $L(\varepsilon,k)\leqq n_1<n_2<\cdots<n_k$,

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