14. On the Propagation of Analyticity for Some Class of Differential Equations with Non-involutive Double Characteristics

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1. Introduction. Let Ω be an open set in $\boldsymbol{R}^{n+1}$ containing the origin, with the coordinates $(x_0, \cdots, x_n)$. We shall consider the differential equation:

(1) $$P(x, D_x)u(x)=f(x), \quad f(x)\in \mathcal{A}(\Omega), \quad u(x)\in \mathcal{D}'(\Omega),$$

where $D_x=-i\partial/\partial x$, and $P(x, D_x)$ is a second order linear differential operator with analytic coefficients in Ω.

Let $p_2(x,\xi)$ be the principal symbol of $P(x, D_x)$. For k, l satisfying $k+l<n$ we put $(x',\xi')=(x_1,\cdots,x_k;\xi_1,\cdots,\xi_k)$, $(x'',\xi'')=(x_{k+1},\cdots,x_{k+l};\xi_{k+1},\cdots,\xi_{k+l})$. We assume the following hypotheses:

(i) *p_2 has the form*

(2) $$p_2(x,\xi)=\xi_0^2-a(x,\xi)+b(x,\xi),$$

where a, b are real valued and non-negative functions independent of ξ_0 and homogeneous of degree 2 with respect to ξ.

(ii) *$a(x,\xi)$ (resp. $b(x,\xi)$) vanishes exactly of order 2 on $\xi'=0$ (resp. $x''=\xi''=0$) in a conic neighborhood of $(0;0,\cdots,0,1)$ in $T^*\Omega$.*

From (i), (ii) we can see that $p_2(x,\xi)$ has doubly characteristic points on $\Lambda=\{(x,\xi)\,|\,x''=\xi_0=\xi'=\xi''=0\}$ which is a non-involutive submanifold of $T^*\Omega$. We shall investigate the propagation of analyticity of a solution $u(x)$ of (1) along the leaf $\Gamma=\{(x,\xi)\,|\,x_i=0,\ k+1\leqq i\leqq n,\ \xi_i=0,\ 0\leqq i\leqq n-1,\ \xi_n=1\}$ of Λ. We regard $(x_0,\cdots,x_k)$ as the coordinates of Γ and $(x_0,\cdots,x_k;\tilde{\xi}_0,\cdots,\tilde{\xi}_k)$ as those of $T^*\Gamma$. In order to state our theorem we introduce the function $q(x_0,x';\tilde{\xi}_0,\tilde{\xi}')$ on $T^*\Gamma$ as follows:

(3) $$q(x_0,x';\tilde{\xi}_0,\tilde{\xi}')=\tilde{\xi}_0^2-\sum_{1\leqq i,j\leqq k}\tilde{\xi}_i\tilde{\xi}_j\partial_{\xi_i}\partial_{\xi_j}a(x_0,x',0;0,\cdots,0,1)/2.$$

Let Σ_t be the subset of Γ defined as the intersection of the hypersurface $S_t=\{(x_0,x')\,|\,x_0=t\}$ and the projection to Γ of the integral curves of

(4) $$H_q=2\tilde{\xi}_0\frac{\partial}{\partial x_0}-\frac{\partial q}{\partial \tilde{\xi}'}\frac{\partial}{\partial x'}+\frac{\partial q}{\partial x'}\frac{\partial}{\partial \tilde{\xi}'},$$

in $T^*\Gamma$ through a point $(0;\tilde{\xi}_0,\tilde{\xi}')$ such that $q(0;\tilde{\xi}_0,\tilde{\xi}')=0$. Further let Ω_t be the connected component of $S_t\backslash\Sigma_t$ which is relatively compact. Then we have,

Theorem. *Let t_0, t_1 be positive real numbers such that $t_0\geqq t_1$ and $\bigcup_{0\leqq t\leqq t_0}\Omega_t\subset\Omega$, and assume that a solution $u(x)$ of (1) satisfies*

(5) $$WF_a(u)\cap\Sigma_{t_0}=\varnothing,$$