107. Interaction of Two Nonlinear Waves at the Boundary

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§0. Introduction. In this paper, we will consider the following nonlinear mixed probrem:

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)u=f\left(\frac{\partial u}{\partial y}\right)$$
$$\text{in } \Omega_M^+=\{(t,x,y)\in \boldsymbol{R}^3\,;\,0<y,\ |t|<M\}$$
$$u|_{y=0}=0$$

where f is a smooth function which will be chosen later and M a positive number.

On the semilinear Cauchy problem, Rauch and Reed [5] have shown by a simple example that anomalous singularities arise when three characteristic hypersurfaces Σ_1, Σ_2 and Σ_3, carrying progressing waves intersect. On the other hand, Bony [2], [3] and Melrose and Ritter [4] have shown that the phenomena of interactions of singularities do not occur when two hypersurfaces Σ_1 and Σ_2 intersect.

In the case of nonlinear mixed problem, Beals and Métivier [1] have shown that when single characteristic hypersurface hits the boundary transversally, then the solution will be conormal with respect to the union of the surface and the reflected characteristic hypersurface.

We will apply the method of [5] to the nonlinear mixed problem and show by an example that anomalous singularities arise when even two hypersurfaces hit the boundary at the same time.

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§1. Singularities of the solution to a linear problem. In this section, we will estimate from below the singularities of the solution V of the equations

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)V=\chi_{\Gamma^+}$$
$$V|_{y=0}=0$$
$$V|_{t=0}=\left.\frac{\partial V}{\partial t}\right|_{t=0}=0.$$

Here $\Gamma^+=\{(t,x,y)\in \boldsymbol{R}^3\,;\,y>0,\ t>0,\ y<-x+\sqrt{2}\,t,\ y<x+\sqrt{2}\,t\}$ and χ_{Γ^+} is its characteristic function.

Proposition 1. *Sing supp V contains the forward light cone C_0^+ with vertex at origin.*

Proof. We will consider sing supp $V\cap\{t=1\}$. For general t, one can