101. Sugawara Operators and their Applications to Kac-Kazhdan Conjecture

By Takahiro HAYASHI
Department of Mathematics, Nagoya University

(Communicated by Kunihiko KODAIRA, M. J. A., Nov. 12, 1987)

§1. **Introduction.** Let $\mathfrak{g}=\mathfrak{n}_-\oplus\mathfrak{h}\oplus\mathfrak{n}_+$ be an affine Kac-Moody Lie algebra of type $X_l^{(1)}$ and its triangular decomposition. A $\mathfrak{g}$-module V is called a highest weight module (HWM) with highest weight (HW) $\lambda\in\mathfrak{h}^*$ if V is generated by a vector $v_\lambda\in V$ such that

$$hv_\lambda=\langle\lambda, h\rangle v_\lambda \quad (h\in\mathfrak{h}) \quad \text{and} \quad \mathfrak{n}_+v_\lambda=0.$$

We call v_λ the highest weight vector of V. There exists the unique $\mathfrak{n}_-$-free HWM $M(\lambda)$ with HW λ. We call it the Verma module of $\mathfrak{g}$ with HW λ. There also exists the unique irreducible HWM with HW λ and we denote it by $L(\lambda)$.

For an HWM V and $\mu\in\mathfrak{h}^*$, set $V_\mu=\{v\in V\,|\,hv=\langle\mu,h\rangle v\ (h\in\mathfrak{h})\}$. Then V is isomorphic to the direct sum of V_μ's and $\dim V_\mu<\infty$ for each $\mu\in\mathfrak{h}^*$. Hence we can define its formal character by

$$\operatorname{ch} V=\sum_{\mu\in\mathfrak{h}^*}(\dim V_\mu)e^\mu.$$

Here e^μ denotes the formal exponential.

The character of the Verma module is given by

$$\operatorname{ch} M(\lambda)=e^\lambda\prod_{\alpha\in\Delta_+}(1-e^{-\alpha})^{-\dim\mathfrak{g}_\alpha}.$$

where Δ_+ denotes the set of the positive root of $\mathfrak{g}$.

For a dominant integral weight λ, the character of the irreducible HWM $L(\lambda)$ is well known as the celebrated Weyl-Kac character formula. However it is difficult to determine $\operatorname{ch} L(\lambda)$ for general weight λ. V. G. Kac and D. A. Kazhdan [4] proposed a study of the irreducible HWM $L(-\rho)$ and gave a conjecture:

$$\operatorname{ch} L(-\rho)=e^{-\rho}\prod_{\alpha\in\Delta_+^{\mathrm{re}}}(1-e^{-\alpha})^{-1},$$

where ρ is the normalized half sum of the positive roots, and Δ_+^{re} is the set of positive real roots.

We give the affirmative result for this conjecture in a more general situation.

Definition. Let c be the canonical central element of $\mathfrak{g}$ and $g=\langle\rho, c\rangle$ be the dual Coxeter number of $\mathfrak{g}$. For a $\lambda\in\mathfrak{h}^*$ with the level $\langle\lambda, c\rangle=-g$, we say that λ is a *KK-weight* if $\langle\lambda+\rho,\alpha^\vee\rangle\notin\boldsymbol{Z}_{>0}$ for each real positive coroot $\alpha^\vee$.

Remark that $-\rho$ is a *KK*-weight. Then one of our main results is the following.

Theorem A. *Let $\mathfrak{g}$ be an affine Lie algebra of type $A_l^{(1)}$, $B_l^{(1)}$ or $C_l^{(1)}$.*