# 99. An Extension of a Uniform Asymptotic Stability Theorem by Matrosov

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**1. Introduction.** The purpose of this paper is to extend Matrosov's result [2] and to obtain sufficient conditions for uniform asymptotic stability of solutions of ordinary differential equations.

The uniform asymptotic stability theorems for autonomous or periodic differential equations were established by Barbashin and Krasovski [3]. These results were generalized to nonautonomous systems by Matrosov [2], Hatvani [1], [3], etc.

Let us consider the nonautonomous ordinary differential equation

$$(1) \qquad \dot{x}=X(t,x), \qquad (X(t,0)\equiv 0),$$

where $X:\Gamma\to\boldsymbol{R}^n$ is a continuous function, $\Gamma=\{(t,x)\in\boldsymbol{R}^+\times\boldsymbol{R}^n:\|x\|<H\}$ for some $H>0$, $\boldsymbol{R}^+=[0,+\infty)$, and $\|x\|$ is the Euclidean norm of a point $x=(x_1,\cdots,x_n)\in\boldsymbol{R}^n$.

Matrosov [2] gave sufficient conditions for uniform asymptotic stability of zero solution of (1) by using a Liapunov function $V$ which has a negative semi-definite time derivative $\dot{V}_{(1)}$ with respect to (1). In Theorem 1.2 of [2], it is assumed that the function $X$, its partial derivatives $\partial X/\partial t$, $\partial X/\partial x_i$ $(i=1,2,\cdots,n)$, and the partial derivatives of $V$: $\partial V/\partial t$, $\partial V/\partial x_i$, $\partial^2 V/\partial t\partial x_i$, $\partial^2 V/\partial x_i\partial x_j$ $(i,j=1,2,\cdots,n)$ are continuous and bounded.

In the present paper, we remove these assumptions and assume more general conditions, which include the above assumptions as a special case.

**2. Theorems.** For $\varepsilon>0$, let $B_\varepsilon=\{x\in\boldsymbol{R}^n:\|x\|<\varepsilon\}$. The closure of a set $E\subset\boldsymbol{R}^n$ is denoted by $\bar{E}$. A function $a(\cdot)$ is said to belong to class $\mathcal{K}$ if $a(\cdot)$ is a continuous, strictly increasing function on $\boldsymbol{R}^+$ into $\boldsymbol{R}^+$ with $a(0)=0$. Let $A:\Gamma\to\boldsymbol{R}$ be a continuous function which satisfies locally a Lipschitz condition with respect to $x$. The time derivative of $A$ with respect to (1) is defined by

$$\dot{A}_{(1)}(t,x)=\limsup_{h\to 0+}\frac{1}{h}[A(t+h,\ x+hX(t,x))-A(t,x)] \quad ((t,x)\in\Gamma).$$

**Theorem 1.** *Suppose that there exists a continuous function $V:\Gamma\to\boldsymbol{R}^+$ which satisfies locally a Lipschitz condition with respect to $x$. For any $\alpha_1,\alpha_2$ $(0<\alpha_1<\alpha_2<H)$, let $\Lambda(\alpha_1,\alpha_2)=\{x\in\boldsymbol{R}^n:\alpha_1\le\|x\|\le\alpha_2\}$. Suppose that there exists a continuously differentiable function $W:\boldsymbol{R}^+\times\Lambda(\alpha_1,\alpha_2)\to\boldsymbol{R}$ such that for some $a,b\in\mathcal{K}$ the following conditions hold.*

(i) $a(\|x\|)\le V(t,x)\le b(\|x\|)$ *in* $\Gamma$.