97. On a Class of Partially Hypoelliptic Microdifferential Equations

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§1. **Introduction.** We study a class of microdifferential equations with double characteristics which are non-hyperbolic. Explicitly, let M be a real analytic manifold with a complexification X and let P be a microdifferential operator defined in a neighborhood of $\rho_0 \in \dot{T}^*_M X$ $(=T^*_M X \setminus M)$ whose principal symbol is written as

(1) $$p=\sigma(P)=p_1+\sqrt{-1}\,q_1^{2m}\cdot p_2$$

in a neighborhood of ρ_0. Here p_1, p_2 and q_1 are homogeneous holomorphic functions of order 1, 1 and 0 respectively, which are defined in a neighborhood of ρ_0. We assume that p_1, p_2 and q_1 satisfy the following conditions (2)–(6).

(2) p_1, p_2 and q_1 are real valued on $T^*_M X$.

(3) dp_1, dp_2 and ω (the canonical 1-form of $T^*_M X$) are linearly independent at ρ_0.

(4) $\{p_1, p_2\}=0$ if $p_1=p_2=0$ where $\{\cdot, \cdot\}$ denotes Poisson bracket on $T^*_M X$.

(5) $$\{p_1, q_1\}\neq 0 \quad \text{at} \quad \rho_0.$$

(6) $$p_1(\rho_0)=p_2(\rho_0)=q_1(\rho_0)=0.$$

We give a theorem concerning the propagation of singularities of solutions to $Pu=0$ on the regular involutory submanifold

$$\Sigma=\{\rho \in \dot{T}^*_M X\,;\, p_1(\rho)=p_2(\rho)=0\}.$$

Precisely, we will show supp (u) is a union of bicharacteristic leaves of Σ for any $u \in \mathcal{C}_{M,\rho_0}$ satisfying $Pu=0$. Interesting is the fact that P is hypoelliptic in the framework of 2-microlocalization.

§2. **Preliminary.** Let M be a real analytic manifold with a complexification X and Σ be a regular involutory submanifold of $\dot{T}^*_M X$. Take a complexification Λ of Σ in T^*X. Then $\tilde{\Sigma}$ denotes the union of all bicharacteristic leaves of Λ eminated from Σ. On $T^*_{\Sigma}\tilde{\Sigma}$, M. Kashiwara constructed the sheaf $\mathcal{C}^2_{\Sigma}$ of 2-microfunctions along Σ. (See Kashiwara-Laurent [2] for details about $\mathcal{C}^2_{\Sigma}$.) We can study the properties of microfunctions on Σ precisely by $\mathcal{C}^2_{\Sigma}$. Actually, we have the following exact sequences (7) and (8).

(7) $$0\longrightarrow \mathcal{C}_{\tilde{\Sigma}}|_{\Sigma}\longrightarrow \mathcal{B}^2_{\Sigma}\longrightarrow \dot{\pi}_*(\mathcal{C}^2_{\Sigma}|_{T^*_{\Sigma}\tilde{\Sigma}\setminus\Sigma})\longrightarrow 0. \qquad (\dot{\pi}: T^*_{\Sigma}\tilde{\Sigma}\setminus\Sigma\longrightarrow\Sigma.)$$

(8) $$0\longrightarrow \mathcal{C}_M|_{\Sigma}\longrightarrow \mathcal{B}^2_{\Sigma}.$$

Here $\mathcal{C}_{\tilde{\Sigma}}$ is the sheaf of microfunctions along $\tilde{\Sigma}$ and $\mathcal{B}^2_{\Sigma}=\mathcal{C}^2_{\Sigma}|_{\Sigma}$.
Moreover there exists the canonical spectral map

(9) $$Sp^2_{\Sigma}: \pi^{-1}\mathcal{B}^2_{\Sigma}\longrightarrow \mathcal{C}^2_{\Sigma} \qquad (\pi: T^*_{\Sigma}\tilde{\Sigma}\longrightarrow\Sigma),$$