92. Moduli Spaces of B_2-connections over Quaternionic Kähler Manifolds

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(Communicated by Kunihiko KODAIRA, M. J. A., Oct. 12, 1987)

The purpose of this note is to announce our recent results on the moduli space of B_2-connections over a quaternionic Kähler manifold. Let M be a $4n$-dimensional compact, connected quaternionic Kähler manifold of positive scalar curvature, and $p: Z \to M$ the corresponding twistor space (see Salamon [8] for the definition of quaternionic Kähler manifolds and the corresponding twistor spaces). Furthermore, let E be a C^∞ complex vector bundle of rank r over M and h a Hermitian metric on E. In [6], we introduced the notion of B_2-connections on E, which generalizes that of anti-self-dual connections for $n=1$. We now define the sets $\mathcal{C}_B, \mathcal{C}_E$ and $\tilde{\mathcal{C}}_E$ as follows. $\mathcal{C}_B$: the set of all Hermitian, B_2-connections on (E, h) over M, $\mathcal{C}_E$: the set of all Einstein-Hermitian connections on (p^*E, p^*h) over Z, $\tilde{\mathcal{C}}_E$: the set of all Einstein-Hermitian connections D on (p^*E, p^*h) over Z, satisfying the following conditions (a) and (b).

(a) Write D as a sum of $D'+D''$ of its (1, 0)- and (0, 1)- components. (In terms of D'', the vector bundle $(F(=p^*E), p^*h)$ is a holomorphic vector bundle.) Then on each fibre Z_m ($m \in M$) of $p: Z \to M$, the restricted vector bundle $(F|_{Z_m}, p^*h|_{Z_m})$ is a flat holomorphic vector bundlle. (Hence the real structure $\tau: Z \to Z$ (cf. Nitta and Takeuchi [7]) naturally lifts to a C^∞ bundle automorphism $\tau': F \to F$.)

(b) Let $\sigma: F \to F^*$ be the bundle map defined fibrewise by

$$F_z \ni f \longrightarrow \sigma(f) \in F^*_{\tau(z)} \qquad (z \in Z),$$

where $\sigma(f)(g) := (p^*h)(g, \tau'(f))$ for each $g \in F_{\tau(z)}$. Then σ is an anti-holomorphic bundle automorphism.

In [6; (0.2), (4.2)], we obtained the following generalization of a result of Atiyah, Hitchin and Singer [1].

Theorem 1. *The mapping*

$$p^*: \mathcal{C}_B \ni D \longrightarrow p^*D \in \tilde{\mathcal{C}}_E (\subset \mathcal{C}_E)$$

gives a bijective correspondence: $\mathcal{C}_B \simeq \tilde{\mathcal{C}}_E$.

The frame bundle of the Hermitian vector bundle (E, h) can be reduced to a principal $U(r)$-bundle P. Put $U_P := P \times_\theta U(r)$ and $\mathfrak{su}_P := P \times_{Ad'} \mathfrak{su}(r)$, where $\theta: U(r) \to \mathrm{Aut}(U(r))$ is the group conjugation and $Ad': U(r) \to GL(\mathfrak{su}(r))$ is the restriction of the adjoint representation of $U(r)$ to $\mathfrak{su}(r)$. The pull-back $p^*(U_P)$ and $p^*(\mathfrak{su}_P)$ are equal to $(p^*P) \times_\theta U(r)$ and $(p^*P) \times_{Ad'} \mathfrak{su}(r)$, respectively. Let D be a Hermitian B_2-connection on (E, h). Then there is an elliptic complex Σ_D on M, $\mathfrak{su}_P$-valued, introduced by Capria