88. Boundly Factorizable $\mathcal{S}$-indecomposable Semigroups Generated by Two Elements

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1. Introduction. A semigroup S is called *$\mathcal{S}$-indecomposable* if it has no semilattice homomorphic image except the trivial image. In particular S is called *boundly factorizable* if $\bigcap_{n=1}^{\infty} S^n=\emptyset$, equivalently, for every $x \in S$ there is a positive integer n such that $x=x_1\cdots x_m$, $x_i \in S$ $(i=1,\cdots,m)$ implies $m\leq n$. Examples of such a semigroup are idempotent-free commutative archimedean semigroups [4], the semilattice components of a free semigroup [3], and so on. To study a boundly factorizable $\mathcal{S}$-indecomposable semigroup S generated by two elements, we consider S as a homomorphic image of the free semigroup of rank 2. Thus our study is reduced to the study of generating relations. In this paper we will investigate basic relations which produce S. See the commutative case in [2].

2. Preliminaries. A semigroup S is called *finitely factorizable* if every element of S is factorized into the product of elements of S in a finitely many ways. By an *irreducible basis* of S we mean a non-empty subset B of S such that (i) S is generated by B, (ii) if $x \in B$ then $x\neq yz$ for all $y, z \in S$. It is known [5] that if $S\backslash S^2$ generates S then $S\backslash S^2$ is an irreducible basis.

Lemma 1 (Theorem 2.2 in [1]). *If a semigroup S is boundly factorizable, then S has an irreducible basis R.*

From the definition we easily have

Lemma 2. *If S is boundly factorizable, then $x\neq yx$, $x\neq xz$ and $x\neq yxz$ for all $x, y, z \in S$. Hence S has no idempotent and S has no minimal ideal.*

Assume a semigroup S is generated by two elements p, q i.e. $R=\{p, q\}$. Let $C(p)$ and $C(q)$ denote the cyclic subsemigroups of S generated by p and q respectively, and $C(p, q)$ the subsemigroup of S consisting of all elements of S which can be expressed as the product of both p and q. In [3] $C(p)$, $C(q)$ and $C(p, q)$ were called contents.

Lemma 3. [3] *Every content in a semigroup is $\mathcal{S}$-indecomposable.*

Lemma 4. *S is $\mathcal{S}$-indecomposable if and only if $C(p)\cap C(p, q)\neq\emptyset$ and $C(q)\cap C(p, q)\neq\emptyset$.*

Let $o_p(v)$ denote the highest degree of the powers of p in v, for example, if $v=p^3qp^2q^2$, then $o_p(v)=3$ and $o_q(v)=2$.

Lemma 5. *If S is a boundly factorizable $\mathcal{S}$-indecomposable semigroup generated by p and q $(p\neq q)$, then*