85. *Reduced Group C^*-Algebras with the Metric Approximation Property by Positive Maps*

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1. **Introduction.** Choi and Effros [3] and Kirschberg [6] have proved that the nuclearity for a C^*-algebra is equivalent to "the complete positive approximation property". Not all C^*-algebras have the approximation property. In fact, A. Szankowski [8] has proved, that the algebra of all bounded operators $B(H)$ on an infinite dimensional Hilbert space H, does not have the approximation property. It had been believed that every C^*-algebra with the metric approximation property is nuclear. Surprisingly, in 1979, Uffe Haagerup [5] showed an example of a non-nuclear C^*-algebra, which has the metric approximation property. Haagerup's example is the reduced group C^*-algebra $C_r^*(F_2)$ of the free group on two generators F_2. In the sequel, Canniere and Haagerup [1] showed that for any fixed $n \in N$, the identity map of $C_r^*(F_2)$ can be approximated by n-positive finite rank operators on $C_r^*(F_2)$. In this note, we shall show that the identity map of the reduced group C^*-algebras generated by the free product of finite groups with one amalgamated subgroup can be approximated by n-positive maps as well. This is an improvement of our previous result in [4].

2. **Results.** Let $G = A \underset{C}{*} B$ be the free product of two finite groups A and B with one amalgamated subgroup C (cf. [7]). Then there is a tree X on which G acts as follows: Put

$V(X) = (G/A) \cup (G/B)$(disjoint union), the set of vertices of X.

$E(X) = (G/C) \cup \overline{(G/C)}$(disjoint union), the set of edges of X.

The source map $s : G/C \to G/A$ and the range map $r : G/C \to G/B$ are induced by the inclusions $C \to A$ and $C \to B$. An action of G on the tree X is given by $g\cdot(xA) = (gx)A \in V(X)$, $g\cdot(xB) = (gx)B \in V(X)$ and $g\cdot(xC) = (gx)C \in E(X)$ for all g, x in G. Put $P_0 = A \in V(X)$. For g in G, define $\Psi(g) = d(P_0, gP_0)$ to be the distance from P_0 to gP_0. Then Ψ is a length function on G[2], [9] such that $\Psi(g)$ is an even integer for all $g \in G$. Note that edges of X consist of

$$xA \circ\!\xrightarrow{\{xC,\ \overline{xC}\}}\!\circ\, xB \qquad x \in G$$

If $d(P_0, Q)$ is even (resp. odd) for $Q \in V(X)$, then $Q = gA = gP_0$ (resp. $Q = gB$) for some $g \in G$. For s in G and integers k, $l \geqq 0$, put

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