84. On a Generalization of Bochner's Tube Theorem for Generic CR-Submanifolds

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The classical Bochner's tube theorem states that every holomorphic function on a connected tube domain $\boldsymbol{R}^n+i\Omega\subset\boldsymbol{C}^n$ can be extended holomorphically to the convex hull of the tube $\boldsymbol{R}^n+ich(\Omega)$.

H. Komatsu [4] has obtained a simple proof of the local version of this theorem by using Cauchy's integral formula. By making use of the theory of Fourier-Bros-Iagolnitzer transform, M. S. Baouendi and F. Treves [1] have generalized the result above. In particular they have obtained the microlocal version of Bochner's tube theorem for generic CR-manifolds.

In this paper we shall give a simple proof of this result. In the section 1, we formulate Bochner's tube theorem for generic CR-submanifolds by employing the notion of specialization of sheaf of holomorphic functions (cf. [5]). In the section 2 we give the new proof of the theorem by reducing the problem to the totally real case.

1. Statement of the result. Let N be a real analytic submanifold of a complex manifold X. For $p\in N$, we denote by $H_p(N)$ the complex tangent space to N at p. The submanifold N is said to be generic, if for all $p\in N$, $\dim_{\boldsymbol{C}} H_p(N)=\dim_{\boldsymbol{C}} X-\operatorname{codim}_{\boldsymbol{R}} N$. Let us assume hereafter that N is generic.

Let S_NX be the spherical normal bundle $T_NX-\{0\}/\boldsymbol{R}^+$ with the projection $\tau: S_NX\to N$. The disjoint union ${}^{\tilde{N}}X=(X-N)\coprod S_NX$ has the structure of real analytic manifold with boundary S_NX. It is called the real monoidal transform of X with center N.

Let i (resp. $\tilde{i}$) be the embedding $i:N\to X$ (resp. $\tilde{i}:S_NX\to{}^{\tilde{N}}X$) and j (resp. $\tilde{j}$) be the natural inclusion map $j:X-N\to X$ (resp. $\tilde{j}:X-N\to{}^{\tilde{N}}X$). We set $\tilde{\mathcal{A}}_{N|X}=\tilde{i}^{-1}(\tilde{j}_*(j^{-1}\mathcal{O}_X))$.

Recall that a subset V of S_NX is said to be convex if each fiber of τ is convex. For every subset U in S_NX, we denote by $ch(U)$ the smallest convex set containing U.

Theorem (cf. [1]). *Let N be a real analytic submanifold of X. We assume that N is a generic* CR-*submanifold. Let U be an open connected subset of S_NX. Then the following assertions hold.*

(a) $\Gamma(U;\tilde{\mathcal{A}}_{N|X})=\Gamma(ch(U);\tilde{\mathcal{A}}_{N|X})$.

(b) *If* $ch(U)=\tau^{-1}(\tau(U))$, *then* $\Gamma(U;\tilde{\mathcal{A}}_{N|X})=\Gamma(\tau(U);\mathcal{O}_{X|N})$.

We note here that the last statement imply the classical Kneser's theorem [3].

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