82. *Uniqueness in the Characteristic Cauchy Problem under a Convexity Condition*

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We consider the Cauchy problem with characteristic initial surface assuming the coefficients to be analytic. Though the uniqueness does not hold in general for C^∞ or $\mathcal{D}'$ solutions, we can expect it if we impose some convexity condition. We establish such a uniqueness theorem at a doubly characteristic point. The result makes us be able to understand the Trèves' example [6] in a general structure.

1. Result. Let U be a neighborhood of the origin in $\boldsymbol{R}^{n+1}$, $P(x;\partial)=\sum_{|\alpha|\le m} a_\alpha(x)\partial^\alpha$, $x=(x_0,\cdots,x_n)$, and $a_\alpha(x)$ be analytic functions in U. We denote the principal symbol of P by $p_m(x, \sum \xi_i dx_i)$. Let S be a hypersurface defined by $\varphi(x)=0$, where φ is a real-valued analytic function satisfying $\varphi(0)=0$ and $d\varphi\neq 0$ in U.

We assume

(A) $\qquad p_m(x, d\varphi)\equiv 0$ in U, and $dp_m(x, d\varphi)=0$ at $x=0$.

Under this assumption, we define

$$G=\left(\frac{\partial p_m^{(e_i)}(x, d\varphi)}{\partial x_j}(0)\,;\,\begin{matrix} i=0\downarrow n \\ j=0\rightarrow n\end{matrix}\right).$$

Let $\lambda_0,\cdots,\lambda_n$ be the eigen values of this matrix. Besides, we put

$$\mu=\left\{p_{m-1}(x, d\varphi)+\sum_{|\alpha|=2}\frac{1}{\alpha\,!}p_m^{(\alpha)}(x, d\varphi)\partial_x^\alpha\varphi\right\}_{x=0}.$$

Note. 1) These $n+2$ values $\lambda_0,\cdots,\lambda_n,\mu$ are invariant with respect to the change of coordinates.

2) The matrix G has at least one zero eigen value.

3) Let F be the fundamental matrix of p_m at its critical point $(0, d\varphi(0))$. Then, under the assumption (A), the eigen values of F are equal to $\{\pm\lambda_0,\cdots,\pm\lambda_n\}$, where λ_i's are those of G.

Now let k be the number of non-zero eigen values of G. We put the following four conditions:

C.1 $\qquad k\ge 1.$

C.2 Let Λ be the convex hull, on the complex number plane, of non-zero eigen values of G, then $0\notin\Lambda$.

C.3 $\qquad \mu\notin\left\{\sum_{i=0}^{n}\lambda_i\beta_i\,;\,\beta\in\boldsymbol{N}^{n+1}\right\}.$

C.4 There are n real-valued analytic functions $\varphi_i(x)$, $i=1,\cdots,n$, such that $d\varphi, d\varphi_1,\cdots,d\varphi_n$ are linearly independent and that