75. *Euler Factors Attached to Unramified Principal Series Representations*

By Takao WATANABE
Mathematical Institute, Tohoku University

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1. Introduction. Let G be a connected reductive unramified algebraic group defined over a non-archimedean local field F of characteristic zero. We use the same notation as in [3]. We fix a non-degenerate character φ of $U(F)$. For a regular unramified character $\chi \in X_{\mathrm{reg}}(T)$ of $T(F)$, let $\rho(D_\chi)$ be the unique constituent of the unramified principal series representation $I(\chi)$ which has a Whittaker model with respect to φ (see [3] Theorem 2). The purpose of this note is to give a construction of an Euler factor attached to $\rho(D_\chi)$. A detailed account will be given elsewhere.

Since the minimal splitting field E of G is unramified over F, the Galois group Γ of E over F is cyclic. Let σ be a generator of Γ. Let $({}^LG^0, {}^LB^0, {}^LT^0)$ be a triple defined over the complex number field $\boldsymbol{C}$ which is dual to the triple (G, B, T). Let ${}^LG = {}^LG^0 \rtimes \Gamma$ be the finite Galois form of the L-group of G ([1]) and $X^*({}^LT^0)$ the character group of ${}^LT^0$. For $\gamma \in \Gamma$, $g \in {}^LG^0$ and $\lambda \in X^*({}^LT^0)$, the transform of g (resp. λ) by γ is denoted by ${}^\gamma g$ (resp. ${}^\gamma\lambda$). Let $\mathcal{R}({}^LG^0)$ (resp. $\mathcal{R}({}^LG)$) be the set of equivalence classes of finite dimensional irreducible representations of ${}^LG^0$ (resp. ${}^LG$).

2. The parametrization of $\mathcal{R}({}^LG)$. Let Λ be the set of dominant weights in $X^*({}^LT^0)$. Note that Λ is Γ-invariant. Let Λ/Γ be the set of Γ-orbits in Λ and $[\lambda]$ the Γ-orbit of $\lambda \in \Lambda$. For $[\lambda] \in \Lambda/\Gamma$, $e([\lambda])$ denotes the cardinality of $[\lambda]$. By the classical theory of Cartan and Weyl, there exists a bijection $R^\sim : \Lambda \to \mathcal{R}({}^LG^0)$ such that, for $\lambda \in \Lambda$, each representative of $R^\sim(\lambda)$ has the highest weight λ. For $\lambda \in \Lambda, \gamma \in \Gamma$ and a representative $R(\lambda)$ of $R^\sim(\lambda)$, we define the representation ${}^\gamma R(\lambda)$ of ${}^LG^0$ by ${}^\gamma R(\lambda)(g) = R(\lambda)({}^\gamma g)$, $g \in {}^LG^0$. Then ${}^\gamma R(\lambda)$ has the highest weight ${}^\gamma\lambda$. Thus we can take representatives $R(\lambda)$ of equivalence classes $R^\sim(\lambda)$ satisfying the following relation:

$$R({}^{\sigma^k}\lambda) = {}^{\sigma^k}R(\lambda) \quad \text{for any } \lambda \in \Lambda, \quad k = 0, 1, \cdots, e([\lambda]) - 1.$$

For $\lambda \in \Lambda$, the representation space of $R({}^\gamma\lambda)$, $\gamma \in \Gamma$ is denoted by $V_{[\lambda]}$. Hereafter, we fix a set of such representatives $\{(R(\lambda), V_{[\lambda]}) \mid \lambda \in \Lambda\}$.

We fix an orbit $[\lambda] \in \Lambda/\Gamma$ and put $e = e([\lambda])$. Let $\mathrm{Hom}_{{}^LG^0}(R(\lambda), {}^{\sigma^e}R(\lambda))$ be the space of intertwining operators of $R(\lambda)$ into ${}^{\sigma^e}R(\lambda)$. This space is considered as a one dimensional subspace of $\mathrm{End}(V_{[\lambda]})$. Let $V^\lambda_{[\lambda]}$ be the common highest weight space of $R(\lambda)$ and ${}^{\sigma^e}R(\lambda)$. Then there exists a unique element $Q_{[\lambda]} \in \mathrm{Hom}_{{}^LG^0}(R(\lambda), {}^{\sigma^e}R(\lambda))$ such that the restriction of $Q_{[\lambda]}$ to $V^\lambda_{[\lambda]}$ gives the identity map of $V^\lambda_{[\lambda]}$. Put $A_{[\lambda]} = \{\zeta^k_{|\Gamma|/e} \cdot Q_{[\lambda]} \mid k = 1, 2, \cdots, |\Gamma|/e\}$, where $\zeta_{|\Gamma|/e} = \exp(2\pi\sqrt{-1}\,e/|\Gamma|)$. Since one has $\mathrm{Hom}_{{}^LG^0}(R({}^\gamma\lambda), {}^{\sigma^e}R({}^\gamma\lambda)) = \mathrm{Hom}_{{}^LG^0}(R(\lambda), {}^{\sigma^e}R(\lambda))$