74. *On Subcommutative Rings*

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Introduction. R denotes an associative ring, not necessarily with identity. In [1] Barbilian defines R to be subcommutative if $Rx \subseteq xR$ for all $x \in R$, and in [6] Reid defines R to be subcommutative if $xR \subseteq Rx$ for all $x \in R$. The case $xR = Rx$ for all $x \in R$ implies R is a duo ring i.e. every one-sided ideal of R is a two-sided ideal (see [7]). Whether one prefers the concept of *left* subcommutativity (Reid) or the concept of *right* subcommutativity (Barbilian) seems to be really immaterial. For on the one hand, theorems may be proved from the side preferred and they follow by symmetry from the other; and on the other hand R is right subcommutative iff the opposite ring of R is left subcommutative. In this paper we examine connections between subcommutativity and related concepts in both the unital and non-unital cases. The results are somewhat scattered, but they touch upon several interesting classes of rings. *Subcommutative* will mean right subcommutative, and the word *ideal* without modifier will mean two-sided ideal. We will work on the right.

Subcommutativity and reflexivity. We require concepts of the following kind: Call a right ideal I of R *reflexive* [5] if $xRy \subseteq I$ implies $yRx \subseteq I$ where $x, y \in R$, and assign the term *completely reflexive* [5] to those I for which $xy \in I$ implies $yx \in I$.

Definition. *A right ideal I of R is called quasi-reflexive if whenever X and Y are right ideals of R with $XY \subseteq I$ then $YX \subseteq I$.*

One easily sees that a quasi-reflexive ideal is two-sided. In the unital case the concepts of reflexivity and quasi-reflexivity coincide [5, proposition 2.3]. Complete reflexivity implies quasi-reflexivity. We write $(a)_r$ for the principal right ideal generated by $a \in R$. Then standard arguments yield the following

Lemma. *A right ideal I of R is quasi-reflexive iff $(x)_r(y)_r \subseteq I$ implies $(y)_r(x)_r \subseteq I$ where $x, y \in R$.*

Any prime (semi-prime) ideal of R is quasi-reflexive. Hence the intersection of any set of prime (semi-prime) ideals is quasi-reflexive. This implies that any ideal in a (von Neumann) regular ring is quasi-reflexive. We also note the following: R subcommutative implies R is right duo (i.e. every right ideal of R is two-sided), consequently $(a) = (a)_r$. This fact establishes one part of

Proposition 1. *Let R be subcommutative. Then an ideal I of R is completely reflexive iff it is quasi-reflexive. Moreover, the subset of nil-*