

### 73. Euler Number of Moduli Spaces of Instantons

By Mikiō FURUTA

Department of Mathematics, University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 14, 1987)

**1. Introduction.** Let  $S^4$  be the 4-dimensional sphere with the standard Riemannian metric,  $P_k$  a principal  $SU(2)$ -bundle over  $S^4$  with  $c_2(P_k) = k$  ( $k > 0$ ), and  $P'_k = P_k / \{\pm 1\}$  the principal  $SO(3) (= SU(2) / \{\pm 1\})$ -bundle over  $S^4$  with  $p_1(P'_k) = -4k$ . We denote by  $M_k$  the moduli space of anti-instantons on  $P_k$  (or  $P'_k$ ). It is known that  $M_k$  has a natural structure of  $8k-3$  dimensional smooth manifold [3]. There are explicit descriptions of  $M_k$  [1] [2] [6], but not so much is known about the topology of  $M_k$ . S. K. Donaldson [6] and C. H. Taubes proved that  $M_k$  is connected. J. Hurtubise [10] proved that  $\pi_1(M_k) = 0$  if  $k$  is odd, and  $\pi_1(M_k) = \mathbf{Z}/2$  if  $k$  is even.

It seems that some aspects of the topology of  $M_k$  is related to some profound properties of 4-dimensional smooth manifolds. In a sense, Donaldson's works in [5] and [7] about intersection forms of 4-manifolds are based on the fact that  $M_1$  is diffeomorphic to open 5-disk.

The purpose of the present note is to announce our results about the Euler number of  $M_k$ .

**2. Statement of the main results.** Our first result is:

**Theorem 1.** *The Euler number  $\chi(M_k)$  is equal to the number  $d(k)$  of divisors of  $k$ .*

The orientation preserving isometry group  $SO(5)$  of  $S^4$  acts on  $M_k$  naturally [3]. Let  $T = SO(2) \times SO(2)$  be the maximal torus of  $SO(4) (\subset SO(5))$ , and  $M_k^T = \{[A] \in M_k; g[A] = [A] \text{ for any } g \in T\}$  the fixed point set. We reduce Theorem 1 to the following Theorem 2.

**Theorem 2.** *The number of the connected component of  $M_k^T$  is equal to  $d(k)$ , and each component is diffeomorphic to  $\mathbf{R}$ . Precisely, the number of lifts of  $T$ -action on  $P'_k$  is equal to  $d(k)$ , and our result is that for each lifted action, the moduli space of  $T$ -invariant anti-instanton on  $P'_k$  is diffeomorphic to  $\mathbf{R}$ .*

We can apply Theorem 2 to get some topological results [8].

**3. Outline of the proof.** Donaldson [6] showed that the moduli space of (framed) anti-instantons is identified with the moduli space of (framed) holomorphic vector bundle over  $\mathbf{C}P^2 = \mathbf{C}^2 \cup \ell^\infty$  with rank=2 and trivial on the line  $\ell^\infty$ . To prove Theorem 2, we investigate  $T$ -equivariant holomorphic bundles over  $\mathbf{C}P^2$ . Here we regard  $T$  as the maximal torus of  $SL(2, \mathbf{C})$ . It could be possible to use the explicit description of  $M_k$ . To derive Theorem 1 from Theorem 2, we first show the following.

**Lemma 3.** *Let  $S^1$  be a generic 1-dimensional connected subgroup of*