72. Systems of Microdifferential Equations with Involutory Double Characteristics

Propagation Theorem for Sheaves in the Framework of Microlocal Study of Sheaves

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§1. **Introduction.** We study a class of microdifferential equations with double involutory characteristics. Explicitly, let M be a real analytic manifold with a complex neighborhood X and let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$ module defined in a neighborhood of $\rho_0 \in T^*_M X \backslash M$. (See M. Sato *et al.* [4] and P. Schapira [5] for $\mathcal{E}_X$.) We assume that the characteristic variety of $\mathfrak{M}$ is written in a neighborhood of ρ_0 as

(1) $$ch(\mathfrak{M}) = \{\rho \in T^*X\, ;\, p(\rho)=0\}$$

by a homogeneous holomorphic function p defined in a neighborhood of ρ_0. Here p satisfies the following conditions (2), (3) and (4).

(2) p is real valued on $T^*_M X$.

(3) $\Sigma = \{\rho \in T^*_M X \backslash M\, ;\, p(\rho)=0,\ dp(\rho)=0\}$ is a regular involutory submanifold of $T^*_M X$ of codimension 2 through ρ_0.

(4) $\mathrm{Hess}\,(p)(\rho)$ has rank 1 if $\rho \in \Sigma$.

In §5, we give a propagation theorem of sheaves in the framework of Microlocal Study of Sheaves due to M. Kashiwara and P. Schapira [2], which will play a powerful role in studying the propagation of singularities for microdifferential systems.

§2. **Notation.** To state the results, we give some prerequisites about 2-microfunctions.

Let Λ be a complexification of Σ in T^*X. Then $\tilde{\Sigma}$ denotes the union of all bicharacteristic leaves of Λ issued from Σ. M. Kashiwara introduced the sheaf $\mathcal{C}^2_\Sigma$ of 2-microfunctions along Σ on $T^*_\Sigma \tilde{\Sigma}$. By $\mathcal{C}^2_\Sigma$, we can study the properties of microfunctions on Σ precisely. Actually, we have exact sequences

(5) $$0 \longrightarrow \mathcal{C}_{\tilde{\Sigma}}|_\Sigma \longrightarrow \mathcal{B}^2_\Sigma \longrightarrow \pi_{\Sigma *}(\mathcal{C}^2_\Sigma|_{T^*_\Sigma \tilde{\Sigma} \backslash \Sigma}) \longrightarrow 0 \qquad (\pi_\Sigma : T^*_\Sigma \tilde{\Sigma} \backslash \Sigma \longrightarrow \Sigma)$$

and

(6) $$0 \longrightarrow \mathcal{C}_M|_\Sigma \longrightarrow \mathcal{B}^2_\Sigma.$$

Here $\mathcal{B}^2_\Sigma = \mathcal{C}^2_\Sigma|_\Sigma$ and $\mathcal{C}_{\tilde{\Sigma}}$ is the sheaf of microfunctions along $\tilde{\Sigma}$. Moreover, we have a canonical spectral map

(7) $$Sp^2_\Sigma : \pi_\Sigma^{-1}(\mathcal{C}_M|_\Sigma) \longrightarrow \mathcal{C}^2_\Sigma,$$

by which we define the 2-singular spectrum for $u \in \mathcal{C}_M|_\Sigma$ as

(8) $$SS^2_\Sigma(u) = \mathrm{supp}\,(Sp^2_\Sigma(u)).$$

We can identify