70. *Some Structure Theorems for ω-Stable Groups*

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§ 1. **Introduction.** An ω-stable group is a structure G whose complete theory in the first order language of G, $L(G)$, is ω-stable and the group operation on G is 0-definable in $L(G)$. In this paper we study some structure theorems for ω-stable groups. One of the earliest results in this subject shown by Baldwin and Saxl [1] is that a locally nilpotent (ω-)stable group is solvable. The point of the proof is that every (ω-)stable group G satisfies the minimal condition on centralizers, i.e., G is an $\mathcal{M}_c$-group.

In a previous paper [7] we studied local properties of ω-stable groups of finite Morley rank. In that article we showed that a locally solvable (locally nilpotent) ω-stable group of finite Morley rank is solvable (nilpotent-by-finite). Recently we realized that these results hold for CZ-groups satisfying the maximal condition on closed connected subgroups. A CZ-group is a group G which carries a T_1-topology satisfying the minimal condition on closed sets, such that for each $a \in G$ the following maps from G to itself are continuous:

$$x \mapsto xa, \quad x \mapsto ax, \quad x \mapsto x^{-1}, \quad x \mapsto x^{-1}ax.$$

It is known that every CZ-group is an $\mathcal{M}_c$-group. CZ-groups were introduced by Kaplansky [5] as an abstraction of linear groups. On the other hand, it is well-known that ω-stable groups of finite Morley rank are quite similar to linear groups over algebraically closed fields. Hence it is reasonable that these two classes of groups share the same structure theorems.

In [4], Higgins proved that a locally supersolvable CZ-group satisfying the maximal condition on closed connected subgroups is nilpotent-by-finite, and hence hypercyclic. Our result is the same vein.

Theorem. *A locally supersolvable ω-stable group of finite Morley rank is nilpotent-by-finite, and hence hypercyclic.*

Most of the proof of the original theorem for CZ-groups go through in our context. Hence our main interest is not the particular results which are proved but some behaviors of model closures in ω-stable groups.

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§ 2. **Model closures.** Our notations are standard. Let $\mathcal{A}$ and $\mathcal{B}$ be classes of groups. A group is said to be $\mathcal{A}$-by-$\mathcal{B}$ if G has a normal subgroup N such that $N \in \mathcal{A}$ and $G/N \in \mathcal{B}$. A group G is locally $\mathcal{A}$ if every finitely generated subgroup of G is in $\mathcal{A}$. In this note "definable" always means definable with parameters.