8. *Connections for Vector Bundles over Quaternionic Kähler Manifolds*

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The purpose of this note is to announce our recent results on quaternionic Kähler manifolds (see Salamon [5] for definition of quaternionic Kähler manifolds). Let M be a $4n$-dimensional connected quaternionic Kähler manifold with the corresponding twistor space $p: Z \to M$ (cf. [5]). Furthermore, let $\boldsymbol{H}$ be the skew field of quaternions. Then the $Sp(n)\cdot Sp(1)$-module $\wedge^2 \boldsymbol{H}^n$ is a direct sum $N_2' \oplus N_2'' \oplus L_2$ of its irreducible submodules N_2', N_2'', L_2, where N_2' (resp. L_2) is the submodule fixed by $Sp(n)$ (resp. $Sp(1)$) and for $n=1$, we have $N_2''=\{0\}$. Hence, the vector bundle $\wedge^2 T^*M$ is written as a direct sum $A_2' \oplus A_2'' \oplus B_2$ of its holonomy-invariant subbundles in such a way that A_2', A_2'', B_2 correspond to N_2', N_2'', L_2, respectively. Now, let V be a vector bundle over M.

Definition 1. A connection for V is called an A_2'-*connection* (resp. B_2-*connection*) if the corresponding curvature is an $\mathrm{End}(V)$-valued A_2'-form (resp. B_2-form).

First, we have:

Theorem A (cf. [3]). *All A_2'-connections and also all B_2-connections are Yang-Mills connections.*

Let $\rho: Sp(n) \to GL(2n; \boldsymbol{C})$ be the standard representation of $Sp(n)$. Recall that $Sp(1)=\{h \in \boldsymbol{H} \mid |h|=1\}$. Furthermore, let K' (resp. K'') be the $\boldsymbol{C}$-vector space $\boldsymbol{C}^{2n}$ (resp. $\boldsymbol{C}^2$ $(=\boldsymbol{H})$) endowed with the $Sp(n)$-action (resp. $Sp(1)$-action) defined by

$$Sp(n) \times \boldsymbol{C}^{2n} \ni (g, f) \longrightarrow \rho(g)\cdot f \in \boldsymbol{C}^{2n},$$
$$(\text{resp. } Sp(1) \times \boldsymbol{H} \ni (u, f) \longrightarrow f\cdot u^{-1} \in \boldsymbol{H}).$$

Then the complexification $\boldsymbol{H}^n \otimes_{\boldsymbol{R}} \boldsymbol{C}$ of the $Sp(n)\cdot Sp(1)$-module $\boldsymbol{H}^n$ is naturally identified with $K' \otimes_{\boldsymbol{C}} K''$. Let r be an integer with $r \geqq 2$. Since the submodule $\wedge^r K' \otimes_{\boldsymbol{C}} S^r K''$ of the $Sp(n)\cdot Sp(1)$-module $\wedge^r (K' \otimes_{\boldsymbol{C}} K'')$ $(=\wedge^r (\boldsymbol{H}^n \otimes_{\boldsymbol{R}} \boldsymbol{C}))$ is just $N_r^{\boldsymbol{C}}$ $(=N_r \otimes_{\boldsymbol{R}} \boldsymbol{C})$ for some suitable $Sp(n)\cdot Sp(1)$-module N_r, we have a natural decomposition $\wedge^r \boldsymbol{H}^n = N_r \oplus L_r$ for some complementary $Sp(n)\cdot Sp(1)$-module L_r of N_r in $\wedge^r \boldsymbol{H}^n$ (cf. [3]). Therefore, the vector bundle $\wedge^r T^*M$ is expressed as a direct sum $A_r \oplus B_r$ of subbundles A_r, B_r corresponding to N_r, L_r, respectively. We denote by $\pi^r: \wedge^r T^*M$ $(=A_r \oplus B_r)$ $\to A_r$ the natural projection to the first factor. Then from a theorem of Salamon [6], one easily obtains the following:

Theorem B (cf. [3]). *Assume that ∇ is a B_2-connection on V. Then the following is an elliptic complex:*