63. *Casson's Invariant for Homology 3-Spheres and Characteristic Classes of Surface Bundles*

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1. Introduction. Let $V=S^1\times D^2$ be a (framed) solid torus and choose two disjoint embedded discs D_-, D_+ in ∂V. Define an oriented 3-dimensional handlebody H_g of genus g by $H_g=V_1\natural\cdots\natural V_g$ (g-copies of V) where D_- of V_i is attached to D_+ of V_{i-1}. We denote Σ_g for ∂H_g which has an embedded disc $D^2=D_+$ of V_g. Let $\mathcal{I}_{g,1}$ be the Torelli group of Σ_g rel. D^2. Now let ι_g be a diffeomorphism of Σ_g defined as $\iota_g=\prod_{i=1}^g \rho_i$ where $\rho_i=\varphi_l\varphi_m\varphi_l$, φ_l and φ_m being, respectively, the Dehn twist on the longitude and meridian curves of the framed torus ∂V_i, so that $H_g\bigcup_{\iota_g}(-H_g)=S^3$. For each element $\varphi\in\mathcal{I}_{g,1}$, the manifold $M(\varphi)=H_g\bigcup_{\iota_g\varphi}(-H_g)$ is an oriented homology 3-sphere and we have the Casson invariant $\lambda(M(\varphi))\in \boldsymbol{Z}$. Thus we have a map $\lambda:\mathcal{I}_{g,1}\to\boldsymbol{Z}$. The purpose of the present note is to announce our result concerning the map λ. Briefly speaking we have shown that the Casson invariant is a kind of secondary invariant associated with the characteristic classes of surface bundles introduced in [7]. As a result we have obtained an alternative definition of λ (see Theorems 6 and 7).

2. Johnson's homomorphisms. Let $x_1,\cdots,x_g,y_1,\cdots,y_g$ be the symplectic basis of $H=H_1(\Sigma_g;\boldsymbol{Z})$ such that x_i and y_i are represented by the longitude and meridian of ∂V_i, respectively. Consider the basis $x_i\wedge y_j$ $(i,j=1,\cdots,g)$, $x_i\wedge x_j$ $(i<j)$, $y_i\wedge y_j$ $(i<j)$ of $\wedge^2 H$ and write t_i $(i=1,\cdots,\binom{2g}{2})$ for these elements (in any order). Let T be the submodule of $\wedge^2 H\otimes\wedge^2 H\subset\wedge^2 H\otimes H^2$ generated by $t_i\otimes t_i$ and $t_i\otimes t_j+t_j\otimes t_i$ $(i\neq j)$. Hereafter we simply write $t_i\leftrightarrow t_j$ for $t_i\otimes t_j+t_j\otimes t_i$. Let $\bar{T}$ be the image in $(\wedge^2 H\otimes H/\wedge^3 H)\otimes H$ of T under the projection $\wedge^2 H\otimes H^2\to(\wedge^2 H\otimes H/\wedge^3 H)\otimes H$. Then we have Johnson's homomorphisms:

$$\tau_2:\mathcal{I}_{g,1}\longrightarrow\wedge^3 H$$
$$\tau_3:\mathcal{K}_{g,1}\longrightarrow\bar{T}$$

where $\mathcal{K}_{g,1}$ is the subgroup of $\mathcal{I}_{g,1}$ generated by Dehn twists on bounding simple closed curves (see [4], [5], [6], [9] for details). Define a homomorphism $\theta_0: T\to\boldsymbol{Z}$ by requiring the value of it on each element of the basis of T described above as $\theta_0(x_i\wedge x_j\leftrightarrow y_i\wedge y_j)=1$ and θ_0 (other element)$=0$.

3. Characteristic classes of surface bundles. Here we begin by briefly recalling several results from our previous papers [7], [8], [9]. Let $\mathcal{M}_{g,1}$ be the mapping class group of Σ_g relative to D^2 and let $e_1\in H^2(\mathcal{M}_{g,1};\boldsymbol{Z})$ be the first characteristic class of surface bundles. We constructed a crossed homomorphism $k:\mathcal{M}_{g,1}\to H$, which is uniquely defined up to