51. The Steffensen Iteration Method for Systems of Nonlinear Equations. II

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1. Introduction. In generalizing the Aitken δ^2-process in one dimension to the case of n-dimensions, Henrici [1, p. 116] has considered a formula, which is called the Aitken-Steffensen formula. In [2], we have studied the above Aitken-Steffensen formula for systems of nonlinear equations and shown [2, Theorem 2]. Moreover, in [3], we have considered a method of iteration for the above systems, which is often called the Steffensen iteration method, and shown [3, Theorem 1]. [3, Theorem 1] improves the result of [2, Theorem 2].

We have given the proof of [3, Theorem 1], in which the Sherman-Morrison-Woodbury formula [3, Lemma 4] is used only to determine $(\Delta^2 X(x^{(k)}))^{-1}$, but in this paper we show that the proof can be simplified without using the formula. And we also present a numerical example in order to show the efficiency of the Steffensen iteration method.

2. Statement of results. Let $x=(x_1, x_2, \cdots, x_n)$ be a vector in R^n and D a region contained in R^n. Let $f_i(x)$ $(1\leqq i\leqq n)$ be real-valued nonlinear functions defined on D and $f(x)=(f_1(x), f_2(x), \cdots, f_n(x))$ an n-dimensional vector-valued function. Then we shall consider a system of nonlinear equations

(2.1) $$x=f(x),$$

whose solution is $\bar{x}$. Let $\|x\|$ and $\|A\|$ be denoted by

$$\|x\|=\max_{1\leqq i\leqq n}|x_i| \quad \text{and} \quad \|A\|=\max_{1\leqq i\leqq n}\sum_{j=1}^{n}|a_{ij}|,$$

where $A=(a_{ij})$ is an $n\times n$ matrix. Define $f^{(i)}(x)\in R^n$ $(i=0, 1, 2, \cdots)$ by

$$f^{(0)}(x)=x,$$
$$f^{(i)}(x)=f(f^{(i-1)}(x)) \qquad (i=1, 2, \cdots).$$

Put

$$d^{(0,k)}=x^{(k)}-\bar{x},$$
$$d^{(i,k)}=f^{(i)}(x^{(k)})-\bar{x} \qquad \text{for } i=1, 2, \cdots,$$

and then define an $n\times n$ matrix $D(x^{(k)})$ by

$$D(x^{(k)})=(d^{(0,k)}, d^{(1,k)}, \cdots, d^{(n-1,k)}).$$

Throughout this paper, we shall assume the following five conditions (A.1)–(A.5) which are the same as those of [3].

(A.1) $f_i(x)$ $(1\leqq i\leqq n)$ are two times continuously differentiable on D.

(A.2) There exists a point $\bar{x}\in D$ satisfying (2.1).

(A.3) $\|J(\bar{x})\|<1$, where $J(x)=(\partial f_i(x)/\partial x_j)$ $(1\leqq i, j\leqq n)$.