49. *A Result on the Scattering Theory for First Order Systems with Long-range Perturbations*

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In this report we treat the following differential equation for $\boldsymbol{C}^m$-valued function:

$$D_t u = \Lambda u,$$

where $D_t = (1/i)(\partial/\partial t)$ and

$$\Lambda = E(x)^{-1/2} \sum_{j=1}^{n} A_j D_j E(x)^{-1/2}, \tag{1}$$

A_j's are $m \times m$ constant hermitian matrices, and $E(x)$ is a continuous $m \times m$ hermitian matrix valued function with

$$0 < c_1 I \leq E(x) \leq c_2 I$$

for some constants c_1 and c_2. Λ can be extended to a self-adjoint operator on $\mathcal{H} = L^2(\boldsymbol{R}^n)$. If we substitute $E(x)$ with I in (1), we have a differential operator of constant coefficients:

$$\Lambda^0 = \sum_{j=1}^{n} A_j D_j.$$

Λ^0 can also be extended to a self-adjoint operator on $\mathcal{H}$, and Λ is regarded as a perturbed operator of Λ^0. The main result which we shall report here is the existence theorem of the wave operator between Λ^0 and Λ. We consider the case that the perturbation is long-range. More precisely we assume that

Assumption (E). 1) $E(x) \in C^\infty(\boldsymbol{R}^n)$.

2) $|\partial_x^\alpha (E(x) - I)| \leq (1 + |x|)^{-\delta - |\alpha|}$ for $\delta > 0$ and $|\alpha| \geq 0$.

The operator $W_\pm$ is called the wave operator if the limit

$$W_\pm u = \lim_{t \to \pm\infty} e^{it\Lambda} e^{-it\Lambda^0} u \qquad (u \in \mathcal{H}_{ac}(\Lambda^0)) \tag{2}$$

exists. In the case of the short-range ($\delta > 1$) it is already known that, for wide class of Λ^0, $W_\pm$ exists and is complete (see for example [3]). But it does not exist generally when the perturbation is long-range ($0 < \delta \leq 1$). Then we should consider the modified wave operator. The fundamental problems of the theory of long-range perturbation are the existence and completeness of the modified wave operator. However few works have been treated related to the spectral theory of systems with long-range perturbations. There are only the works related to the limiting absorption principle ([3], [4]). Then unlike the case of the short-range the existence theorem is the first step of this theory.

On Λ^0 we assume the following. We put

$$\Lambda^0(\xi) = \sum_{j=1}^{n} A_j \xi_j \qquad \text{(symbol of } \Lambda^0\text{)}.$$