41. On the Asymptotic Stability of Solutions of a Second Order Nonlinear Differential Equation

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1. Introduction. In this paper we consider the asymptotic stability of the zero solution of the second order nonlinear differential equation

$$(r(t)x')'+f(t, x, x')x'+p(t)g(x)=0. \tag{1}$$

The stability or attractivity properties of second order nonlinear nonautonomous differential equations are discussed by Burton and Grimmer [2], Baker [1], Hatvani [3], and Yamamoto and Sakata [4], etc. In [3], Hatvani established conditions for the (equi- or uniformly) asymptotic stability of the zero solution of ordinary differential equations and gave an application of his theorem to the equation (1) in the case of $f(t, x, x')\equiv f(t)$.

In the present paper, we investigate the (globally) asymptotic stability, (globally) equi-asymptotic stability, and (globally) uniformly asymptotic stability as well as uniform stability of the zero solution of (1) by applying Hatvani's theorem [3] and one of its extensions [5].

2. Theorems. We consider the equation (1) or the equivalent system

$$x'=y, \qquad y'=-\frac{p(t)}{r(t)}g(x)-\frac{r'(t)+f(t, x, y)}{r(t)}y \tag{2}$$

under the following assumption.

Assumption A. a) p and r are continuously differentiable, positive functions on $\boldsymbol{R}^+=[0, +\infty)$.

(b) $f: \boldsymbol{R}^+\times\boldsymbol{R}^2\to\boldsymbol{R}^+$ is continuous.

(c) $g: \boldsymbol{R}\to\boldsymbol{R}$ is a continuous function such that $xg(x)>0$ $(x\neq 0)$.

We shall use the following notations and definitions.

For $x\in\boldsymbol{R}^n$ and $\varepsilon>0$, let $B_n(x, \varepsilon)=\{y\in\boldsymbol{R}^n: \|y-x\|<\varepsilon\}$. The ε-neighborhood of a set $E\subset\boldsymbol{R}^n$ is the set $B_n(E, \varepsilon)=\{x\in\boldsymbol{R}^n: d(x, E)<\varepsilon\}$, where $d(x, E)=\inf\{\|x-y\|: y\in E\}$ is the distance from $x\in\boldsymbol{R}^n$ to E.

A function a is said to belong to the class $\boldsymbol{K}$ $(a\in\boldsymbol{K})$ if a is a continuous, strictly increasing function on $\boldsymbol{R}^+$ into $\boldsymbol{R}^+$ with $a(0)=0$.

A measurable function $\phi: \boldsymbol{R}^+\to\boldsymbol{R}^+$ is said to be integrally positive (see [3]) if $\int_I \phi(t)dt=+\infty$ on every set $I=\bigcup_{m=1}^{+\infty}[\alpha_m, \beta_m]$ such that $\alpha_m<\beta_m<\alpha_{m+1}$, $\beta_m-\alpha_m\geqq\delta>0$ for $m=1, 2, \cdots$. If, in addition, $\alpha_{m+1}-\beta_m\leqq\gamma$ $(m=1, 2, \cdots)$ for some constant $\gamma>0$, ϕ is said to be weakly integrally positive (see [3]).

We say that a function $\xi: \boldsymbol{R}^+\to\boldsymbol{R}^+$ belongs to the class $\boldsymbol{F}$ $(\xi\in\boldsymbol{F})$ (see [3]) if there are two measurable functions $\xi_1, \xi_2: \boldsymbol{R}^+\to\boldsymbol{R}^+$ such that ξ_1 is bounded