39. A Note on a Generalization of a q-series Transformation of Ramanujan

By H. M. SRIVASTAVA*)
Department of Mathematics, University of Victoria,
Victoria, British Columbia V8W 2Y2, Canada

(Communicated by Kôsaku YOSIDA, M. J. A., May 12, 1987)

It is shown how readily a recent generalization of a q-series transformation of Srinivasa Ramanujan would follow as a limiting case of Heine's transformation for basic hypergeometric series. Several interesting consequences of this general result are also deduced.

For real or complex q, $|q|<1$, let

$$(1)\qquad (\lambda;q)_\mu=\prod_{j=0}^{\infty}(1-\lambda q^j)/(1-\lambda q^{\mu+j})$$

for arbitrary λ and μ, so that

$$(2)\qquad (\lambda;q)_n=\begin{cases}1, & \text{if } n=0,\\ (1-\lambda)(1-\lambda q)\cdots(1-\lambda q^{n-1}), & \forall n\in\{1,2,3,\cdots\},\end{cases}$$

and

$$(3)\qquad (\lambda;q)_\infty=\prod_{j=0}^{\infty}(1-\lambda q^j).$$

The q-series transformation

$$(4)\qquad (-bq;q)_\infty\sum_{n=0}^{\infty}\frac{q^{n^2}}{(-bq;q)_n}\frac{\lambda^n}{(q;q)_n}=\sum_{n=0}^{\infty}q^{(1/2)n(n+1)}\left(-\frac{\lambda}{b};q\right)_n\frac{b^n}{(q;q)_n}$$

is stated in Chapter 16 of the Second Notebook of Srinivasa Ramanujan [9, Vol. II, p. 194, Entry 9]. A special case of Ramanujan's identity (4) when $b=1$ was posed as an Advanced Problem by Carlitz [5, p. 440, Equation (1)] who, in fact, proved the general case (4) by using Euler's expansion for $(\lambda;q)_n$ as a polynomial in λ (cf. [6, p. 917]). The identity (4) has received considerable attention in several subsequent works (see, for example, [1], [2], and [8]). In particular, in their excellent memoir [1, pp. 9–10] Adiga *et al.* have presented two interesting proofs of (4). It should be remarked in passing that one of their proofs using Heine's transformation [7, p. 306, Equation (79)] iteratively is essentially equivalent to the earlier proof by Andrews [2, p. 105] who deduced (4) as a limiting case of a result attributed to Rogers.

An interesting generalization of Ramanujan's q-series transformation (4) was given recently by Bhargava and Adiga in the form (cf. [4, p. 339, Equation (3)]; see also [3, p. 14, Equation (4*)]):

---

*) This research was supported, in part, by the Natural Sciences and Engineering Research Council of Canada under Grant A-7353.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 33A30; Secondary 11B65, 05A30.