35. On Automorphism Groups of Compact Riemann Surfaces of Genus 5

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(Communicated by Shokichi IYANAGA, M. J. A., April 13, 1987)

Let X be a compact Riemann surface of genus $g \geqq 2$. A group AG of automorphism of X can be represented as a subgroup $R(X, AG)$ of $GL(g, C)$ as elements of AG operate in the g-dimensional module of abelian differentials on X. The purpose of this paper is to determine in case $g=5$ all subgroups of $GL(g, C)$ which are conjugate to some $R(X, AG)$ for some X and some AG. For the case $g=2, 3, 4$ the same problem was already solved: [3] for the case $g=2$; the result for $g=3, 4$ is not yet published. A more detailed account will be published elsewhere.

§0. **Preliminaries.** Let G be a finite subgroup of $GL(g, C)$ and let H be a non-trivial cyclic subgroup of G. Define two sets $CY(G)$ and $CY(G; H)$ as in [3]. If any element of $CY(G)$ is $GL(g, C)$-conjugate to a subgroup arising from Riemann surfaces of genus g, then we say that G stands the CY-test. Further we define $l(H; G)$ and $RH(G)$ as in [3]. If G stands the CY-test and $l(H; G)$ is a non-negative integer for every element H of $CY(G)$, then we say that G stands the RH-test. Let $RH(G)$ be $[g_0, n; e_1, \cdots, e_r]$ and let Γ be a Fuchsian group $\langle \alpha_1, \beta_1, \cdots, \alpha_{g_0}, \beta_{g_0}, \gamma_1, \cdots \gamma_r \rangle$ with relations $\prod_{j=1}^{r} \gamma_j \cdot \prod_{i=1}^{g_0} [\alpha_i, \beta_i]=1$, $\gamma_1^{e_1}=\cdots=\gamma_r^{e_r}=1$. If we have a surjective homomorphism $\phi: \Gamma \to G$ such that $\sharp\phi(\gamma_j)=e_j$ and $2-2\,\mathrm{Re}\,(\mathrm{tr}\,\phi(\gamma_j))>0$ $(1 \leqq j \leqq r)$, then we say that G stands the EX-test. If G stands the EX-test, then it can be shown that there exists an $R(X, AG)$ which is $GL(5, C)$-conjugate to G by taking a suitable ϕ [1, 2, 4].

Notations. We use following notations for economy of space.

$$A(a,b,c,d,e)=\begin{bmatrix} a &&&& \\ & b &&& \\ && c && \\ &&& d & \\ &&&& e \end{bmatrix}, \quad B(a,b,c,d,e)=\begin{bmatrix} a &&&& \\ & b &&& \\ && c && \\ &&& 0 & d \\ &&& e & 0 \end{bmatrix}, \quad C(a,b,c,d,e)=\begin{bmatrix} a &&&& \\ & 0 & b && \\ & c & 0 && \\ &&& d & 0 \\ &&& 0 & e \end{bmatrix},$$

$$D(a,b,c,d,e)=\begin{bmatrix} 0 & a &&& \\ b & 0 &&& \\ && c && \\ &&& d & \\ &&&& e \end{bmatrix}, \quad E(a,b,c,d,e)=\begin{bmatrix} a &&&& \\ & 0 & 0 & b & 0 \\ & 0 & c & 0 & 0 \\ & d & 0 & 0 & 0 \\ & 0 & 0 & 0 & e \end{bmatrix}, \quad F(a,b,c,d,e)=\begin{bmatrix} a &&&& \\ & 0 & 0 & 0 & b \\ & 0 & 0 & c & 0 \\ & 0 & d & 0 & 0 \\ & e & 0 & 0 & 0 \end{bmatrix},$$

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