1. *Super Oscillatory Integrals and a Path-integral for a Non-relativistic Spinning Particle*

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Introduction. For a given 'Hamiltonian' described by even and odd Grassmann variables (called super Hamiltonian), we 'quantize' it by applying the method of the product integrals. Namely, introducing the supersymmetric version of the oscillatory integrals (super oscillatory integrals, for short) whose phase and amplitude functions are defined by a super Hamiltonian, we prove the convergence of its iterated integrals under suitable conditions by a similar procedure in Kitada [2]. Detailed proof will appear elsewhere.

Result. Let V_N be a vector space over $\boldsymbol{R}$ of dimension N with a positive definite inner product whose orthonormal basis is given by $\{e_j\}_{j=1}^N$ where $N=2l$. We denote by $Cl(V_N)$ the free algebra over $\boldsymbol{C}$ generated by 1 and $\{e_j\}_{j=1}^N$ with relations $e_je_k+e_ke_j=-2\delta_{jk}$ for $j, k=1, 2, \cdots, N$. We prepare another vector space V_{N+2} over $\boldsymbol{C}$ of dimension $N+2$ with a positive definite inner product whose orthonormal basis is given by $\{e_j\}_{j=-1}^N$. Assuming above relations hold for $j, k=-1, 0, 1, \cdots, N$, we define $Cl(V_{N+2})$ analogously as above. In $Cl(V_{N+2})$, putting $\sigma_j=(1/\sqrt{2})(e_{2j}+\sqrt{-1}e_{2j-1})$ and $\bar{\sigma}_j=(1/\sqrt{2})(e_{2j}-\sqrt{-1}e_{2j-1})$ for $j=0, 1, \cdots, l$, we get easily the following Grassmann relations $\sigma_j\sigma_k+\sigma_k\sigma_j=0$, $\bar{\sigma}_j\bar{\sigma}_k+\bar{\sigma}_k\bar{\sigma}_j=0$ and $\sigma_j\bar{\sigma}_k+\bar{\sigma}_k\sigma_j=2\sqrt{-1}\delta_{jk}$ for $j, k=0, 1, \cdots, l$. We denote by $\mathcal{G}r(l+1)$ the set of free algebra over $\boldsymbol{C}$ generated by 1 and $\{\sigma_j\}_{j=0}^l$. Let S be a set of elements of $\mathcal{G}r(l+1)$ represented as $\psi=\sum_{|a|:\text{even}}\psi_a\sigma^a$. Any element $\psi\in S$ is called a spinor. We consider a spin field $\psi=\psi(q)$ on $\boldsymbol{R}^n$, that is, ψ is a section of a bundle $\pi: \mathcal{S}=\boldsymbol{R}^n\times S\to\boldsymbol{R}^n$ represented as $\psi(q)=\sum_{|a|:\text{even}}\psi_a(q)\sigma^a$, for $q\in\boldsymbol{R}^n$. Denote by $\Gamma_0^\infty(\mathcal{S})$ a set of smooth sections on $\mathcal{S}$ with compact support. For $\psi\in\Gamma_0^\infty(\mathcal{S})$, we put $\|\psi\|^2=\sum_{|a|:\text{even}}\|\psi_a\|^2_{L^2(\boldsymbol{R}^n)}$. We denote by $L^2(\mathcal{S})(=\boldsymbol{H})$ the completion of $\Gamma_0^\infty(\mathcal{S})$ with respect to $\|\cdot\|$. Defining a super-space $\boldsymbol{R}^{n,l+1}$ as a set of points with even coordinates $x_1, x_2, \cdots, x_n$ and odd coordinates $\theta_0, \theta_1, \cdots, \theta_l$, we introduce function spaces over $\boldsymbol{R}^{n,l+1}$ as same as those over $\boldsymbol{R}^n$. We define a mapping $\sharp: \Gamma_0^\infty(\mathcal{S})\to C_{0,e}^\infty(\boldsymbol{R}^{n,l+1})$ by $(\sharp\psi)(x,\theta)\ (=f(x,\theta))=\sum_{|a|:\text{even}}\psi_a(x)\theta^a$, where $\psi_a(x)$ is the Grassmann extension of $\psi_a(q)$. Conversely, for any $f(x,\theta)\in C_{0,e}^\infty(\boldsymbol{R}^{n,l+1})$, we put $(\flat f)(q)=f(q,\sigma_0,\cdots,\sigma_l)$. As $\sharp\flat=\mathrm{Id}$ and $\flat\sharp=\mathrm{Id}$, we have a natural identification between $\Gamma_0^\infty(\mathcal{S})$ (resp. $L^2(\mathcal{S})$) and $C_{0,e}^\infty(\boldsymbol{R}^{n,l+1})$ (resp. $L_e^2(\boldsymbol{R}^{n,l+1})$). Now, we may define an action ρ of $Cl(V_N)$ on $\mathcal{S}$ as

$$\rho(e_{2j})=(1/2)(\sigma_0+\sqrt{-1}*\bar{\sigma}_0\,\lrcorner)(\sigma_{2j}+\sqrt{-1}*\bar{\sigma}_{2j}\,\lrcorner)$$

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